Mini Lecture 10.1
Radical Expressions and Functions

Learning Objectives:

1. Evaluate square roots.
2. Evaluate square root functions.
3. Find the domain of square root functions.
4. Use models that are square root functions.
5. Simplify expressions of the form $\sqrt{a^2}$.
6. Evaluate cube root functions.
7. Simplify expressions of the form $\sqrt[3]{a^3}$.
8. Find even and odd roots.
9. Simplify expressions of the form $\sqrt[n]{a^n}$.

Examples:

1. Evaluate the following:

a. $\sqrt{49}$ b. $-\sqrt{25}$ c. $\sqrt{\frac{4}{9}}$

d. $\sqrt{0.0064}$ e. $\sqrt{64+36}$ f. $\sqrt{64}+\sqrt{36}$

2. For each function, find the indicated function value.

a. $f(x)=\sqrt{4x+5}$; $f(5)$ b. $g(x)=\sqrt{4-2x}$; $g(-3)$ c. $h(x)=\sqrt{2x-3}$; $h(9)$

3. Find the domain for each of the following:

a. $f(x)=\sqrt{5x-20}$ b. $g(x)=\sqrt{9-3x}$

4. Simplify each expression.

a. $\sqrt{(-6)^2}$ b. $\sqrt{(x-4)^2}$ c. $\sqrt{81x^8}$ d. $\sqrt{x^2+4x+4}$

5. For each function, find the indicated function value.

a. $f(x)=\sqrt{2x-1}$; $f(5)$ b. $g(x)=\sqrt[3]{x-9}$; $g(10)$

6. Simplify.

a. $\sqrt[3]{-y^3}$ b. $\sqrt[3]{8a^3}$ c. $\sqrt[4]{1}$

d. $-\sqrt[4]{81}$ e. $\sqrt[4]{-256}$ f. $\sqrt[5]{1}$

g. $\sqrt[4]{(x-2)^4}$ h. $\sqrt[5]{(2x+1)^5}$ i. $\sqrt[6]{(-2)^6}$

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Teaching Notes:

- If $b^2 = a$, then b is the square root of a.
- A square root of a negative number is not a real number.
- The symbol "$\sqrt{\ }$" is called a **radical sign**.
- The number under the radical sign is called the **radicand**.
- Together, the radical sign and the radicand are called the **radical expression**.
- A **square root function** is defined by $f(x) = \sqrt{x}$.
- For any real number a, $\sqrt{a^2} = |a|$.
- The **cube root** of a real number a is written $\sqrt[3]{a}$, the $\sqrt[3]{a} = b$ means $b^3 = a$.
- There is a **cube root function** defined by $f(x) = \sqrt[3]{x}$.
- In the radical expression $\sqrt[n]{a}$, the number n is the **index**. If n is an odd number, then the root is called an **odd root**. If n is even, then the root is called an **even root**.
- An even root of a negative number is **not** a real number.

Answers: 1. a. 7 b. –5 c. $\frac{2}{3}$ d. 0.08 e. 10 f. 14 2. a. $f(5) = 5$ b. $g(-3) = \sqrt{10} \approx 3.16$ c. $h(9) = \sqrt{15} \approx 3.87$ 3. a. domain of f is $\{x \mid x \geq 4\}$ or $[4, \infty)$ b. domain of g is $\{x \mid x \leq 3\}$ or $(-\infty, 3]$
4. a. –6 b. $|x - 4|$ c. $9|x^4|$ d. $|x + 2|$ 5. a. $f(5) = 3$ b. $g(10) = 1$ 6. a. $-y$ b. $2a$ c. 1 d. –3
3. not a real number f. 1 g. $|x - 2|$ h. $2x + 1$ i. 2

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Mini Lecture 10.2
Rational Exponents

Learning Objectives:

1. Use the definition of $a^{\frac{1}{n}}$.
2. Use the definition of $a^{\frac{m}{n}}$.
3. Use the definition of $a^{\frac{-m}{n}}$.
4. Simplify expressions with rational exponents.
5. Simplify radical expressions using rational exponents.

Examples:

Use the radical notation to rewrite each expression, then simplify if possible. Assume that all variables represent positive numbers.

1. a. $64^{\frac{1}{2}}$ b. $81^{\frac{1}{4}}$ c. $(-64)^{\frac{1}{3}}$ d. $(16x^4y^2)^{\frac{1}{2}}$

2. a. $16^{\frac{3}{4}}$ b. $(-27)^{\frac{2}{3}}$ c. $-25^{\frac{3}{2}}$ d. $(-9)^{\frac{1}{2}}$

Rewrite each expression with a positive rational exponent. Simplify if possible.

3. a. $8^{-\frac{1}{3}}$ b. $16^{-\frac{3}{4}}$ c. $27^{-\frac{2}{3}}$ d. $(81x^8)^{-\frac{3}{4}}$

Rewrite each radical expression with rational exponents.

4. a. $\sqrt{5}$ b. $\sqrt[3]{30x}$ c. $\sqrt[3]{\frac{2}{3}}$ d. $\sqrt[6]{3xy^5}$

Use properties of rational exponents to simplify each expression. Assume that all variables represent positive numbers.

5. a. $4^{\frac{1}{4}} \cdot 4^{\frac{1}{4}}$ b. $(8^{\frac{1}{3}})^3$ c. $(27a^6b^3)^{\frac{2}{3}}$ d. $x^4y^3 \cdot x^{\frac{3}{4}}y^{\frac{2}{3}}$

Teaching Notes:

- The denominator of a rational exponent is the index of the equivalent radical.
- The numerator of a radical exponent is the power of which the radical is raised.
- Remind students that each base occurs only once in a simplified expression.
- A simplified expression should not contain any negative exponents.
- Remember, any base other than zero raised to the zero power is the number 1. A simplified expression should not have any zero exponents.

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Answers:

1. a. $\sqrt{64}=8$ b. $\sqrt[4]{81}=3$ c. $\sqrt[3]{-64}=-4$ d. $\sqrt{16x^4y^2}=4x^2y$ 2. a. $(\sqrt[4]{16})^3=8$ b. $(\sqrt[3]{-27})^2=9$ c. $-(\sqrt{25})^3=-125$ d. $\sqrt{-9}=$ Not A Real Number 3. a. $\frac{1}{\sqrt[3]{8}}=\frac{1}{2}$ b. $(\frac{1}{\sqrt[4]{16}})^3=\frac{1}{8}$ c. $\frac{1}{(\sqrt[3]{27})^2}=\frac{1}{9}$ d. $\frac{1}{(\sqrt[4]{81x^8})^3}=\frac{1}{27x^6}$ 4. a. $5^{\frac{1}{2}}$ b. $(30x)^{\frac{1}{3}}$ c. $\left(\frac{2}{3}\right)^{\frac{1}{3}}$ d. $(3xy^5)^{\frac{1}{6}}$ 5. a. 2 b. 8 c. $9a^4b^2$ d. $x^{\frac{19}{4}}y^{\frac{11}{3}}$

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Mini Lecture 10.3

Multiplying and Simplifying Radical Expressions

Learning Objectives:

1. Use the product rule to multiply radicals.
2. Use factoring and the product rule to simplify radicals.
3. Multiply radicals and then simplify.

Examples:

1. Use the product rule to multiply.
   a. $\sqrt{3}\cdot\sqrt{7}$ b. $\sqrt{x-3}\cdot\sqrt{x+3}$
   c. $\sqrt[3]{5}\cdot\sqrt[3]{6}$ d. $\sqrt[5]{3x}\cdot\sqrt[5]{4x^2}$

2. If $f(x)=\sqrt{2x^2+8x+8}$, express the function, *f*, in simplified form.

3. Simplify. Assume all variables in a radicand represent positive real numbers and no radicands involve negative quantities raised to even powers.
   a. $\sqrt{90}$ b. $\sqrt[3]{24}$
   c. $\sqrt[4]{243}$ d. $\sqrt{128x^3}$
   e. $\sqrt{x^8y^7z^6}$ f. $\sqrt[3]{16x^{10}y^{15}}$
   g. $\sqrt[5]{32x^9y^7z^{10}}$ h. $\sqrt{24x^2y^5}$

4. Multiply and simplify. Assume all variables in a radicand represent positive real numbers and no radicands involve negative quantities raised to even powers.
   a. $\sqrt{3}\cdot\sqrt{4}$ b. $5\sqrt[3]{2}\cdot 10\sqrt[3]{16}$
   c. $\sqrt[4]{8x^2y^4}\cdot\sqrt[4]{4x^5y^2}$ d. $\sqrt{3x^7}\cdot\sqrt{20x^5}$

Teaching Notes:

- The **product rule for radicals** states: if $\sqrt[n]{a}$ and $\sqrt[n]{b}$ are real numbers then $\sqrt[n]{a}\cdot\sqrt[n]{b}=\sqrt[n]{ab}$.
- A number that is the square of an integer is a **perfect square**.
- A number is a **perfect cube** if it is the cube of an integer.
- A radical of index *n* is **simplified** when its radicand has no factors other than 1 that are perfect *n*th powers.
- For any non-negative real number, *a*, $\sqrt[n]{a^n}=a$.
- Perfect *n*th powers have exponents that are divisible by *n*.

Answers: 1. a. $\sqrt{21}$ b. $\sqrt{x^2-9}$ c. $\sqrt[3]{30}$ d. $\sqrt[5]{12x^3}$ 2. $f(x)=\sqrt{2}\,|x+2|$ 3. a. $3\sqrt{10}$ b. $2\sqrt[3]{3}$ c. $3\sqrt[4]{3}$ d. $8x\sqrt{2x}$ e. $x^4y^3z^3\sqrt{y}$ f. $2x^3y^5\sqrt[3]{2x}$ g. $2xyz^2\sqrt[5]{x^4y^2}$ h. $2xy^2\sqrt{6y}$ 4. a. $2\sqrt{3}$ b. $100\sqrt[3]{4}$ c. $2xy\sqrt[4]{2x^3y^2}$ d. $2x^6\sqrt{15}$

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Mini Lecture 10.4

Adding, Subtracting, and Dividing Radical Expressions

Learning Objectives:

1. Add and subtract radical expressions.
2. Use the quotient rule to simplify radical expressions.
3. Use the quotient rule to divide radical expressions.

Examples:

Add or subtract. Be sure answers are in simplified form.

1. a. $6\sqrt{10}+3\sqrt{10}$ b. $7\sqrt{18}-3\sqrt{18}$ c. $4\sqrt{6}+3\sqrt{6}-\sqrt{6}$

2. a. $4\sqrt{20}+3\sqrt{5}$ b. $3\sqrt{32x}-2\sqrt{18x}$ c. $3\sqrt{48}+2\sqrt{27}$

3. a. $5\sqrt[3]{7}+4\sqrt[3]{7}$ b. $12\sqrt[4]{32}-3\sqrt[5]{32}$ c. $6\sqrt[3]{54x^3}+2x\sqrt[3]{16}$

Simplify using the quotient rule.

4. a. $\dfrac{\sqrt{8x^3}}{\sqrt{2x}}$ b. $\dfrac{\sqrt[3]{40x^5}}{\sqrt[3]{8x^2}}$ c. $\dfrac{\sqrt{25x^6}}{\sqrt{y^{12}}}$

Divide and simplify if possible.

5. a. $\dfrac{\sqrt{100x^5}}{\sqrt{4x^3}}$ b. $\dfrac{\sqrt[3]{40x^6y^5}}{\sqrt[3]{5x^3y^2}}$ c. $\dfrac{5\sqrt{20x^7}}{\sqrt{5x}}$

Teaching Notes:

- The radicand <u>and</u> the index must be the same in order to add or subtract radicals.
- Sometimes it is necessary to simplify radicals first to find out if they can be added or subtracted.
- The quotient rule can be used in two ways: $\dfrac{\sqrt[n]{a}}{\sqrt[n]{b}}=\sqrt[n]{\dfrac{a}{b}}$ or $\sqrt[n]{\dfrac{a}{b}}=\dfrac{\sqrt[n]{a}}{\sqrt[n]{b}}$.

<u>Answers</u>: 1. a. $9\sqrt{10}$ b. $12\sqrt{2}$ c. $6\sqrt{6}$ 2. a. $11\sqrt{5}$ b. $6\sqrt{2x}$ c. $18\sqrt{3}$

3. a. $9\sqrt[3]{7}$ b. cannot be subtracted c. $22x\sqrt[3]{2}$ 4. a. $2x$ b. $x\sqrt[3]{5}$ c. $\dfrac{5x^3}{y^6}$

5. a. $5x$ b. $2xy$ c. $10x^3$

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Mini Lecture 10.5

Multiplying With More Than One Term and Rationalizing Denominators

Learning Objectives:

1. Multiply radical expressions with more than one term.
2. Use polynomial special products to multiply radicals.
3. Rationalize denominators containing one term.
4. Rationalize denominators containing two terms.
5. Rationalize numerators.

Examples:

1. Multiply.

a. $\sqrt{3}(x+\sqrt{6})$ b. $\sqrt[3]{x^2}(\sqrt[3]{x}-\sqrt[3]{4})$

c. $(2\sqrt{3}+3\sqrt{5})(3\sqrt{3}-2\sqrt{5})$ d. $(\sqrt{7}+\sqrt{5})^2$

e. $(\sqrt{6}+\sqrt{3})(\sqrt{6}-\sqrt{3})$ f. $(\sqrt{x}-\sqrt{5})(\sqrt{x}+\sqrt{5})$

2. Rationalize each denominator.

a. $\dfrac{\sqrt{2}}{\sqrt{5}}$ b. $\sqrt[3]{\dfrac{3}{4}}$ c. $\sqrt{\dfrac{3x}{5y}}$

d. $\dfrac{\sqrt[3]{2x}}{\sqrt[3]{y^2}}$ e. $\dfrac{6}{2\sqrt{5}+4}$ f. $\dfrac{3+\sqrt{6}}{\sqrt{6}-\sqrt{2}}$

3. Rationalize the numerator. $\dfrac{\sqrt{x}-4}{\sqrt{x}+2}$

Teaching Notes:

- To multiply radical expressions with more than one term use the distributive property and the FOIL method.
- To rationalize the denominator, multiply the numerator and the denominator by a radical of index *n* that produces a perfect *n*th power in the denominator's radicand.
- Radical expressions that involve the sum and difference of the same two terms are called **conjugates**.
- To rationalize a denominator with two terms and one or more square roots, multiply the numerator and denominator by the conjugate of the denominator.

Answers: 1. a. $x\sqrt{3}+3\sqrt{2}$ b. $x-\sqrt[3]{4x^2}$ c. $-12+5\sqrt{15}$ d. $12+2\sqrt{35}$ e. 3 f. $x-5$

2. a. $\dfrac{\sqrt{10}}{5}$ b. $\dfrac{\sqrt[3]{6}}{2}$ c. $\dfrac{\sqrt{15xy}}{5y}$ d. $\dfrac{\sqrt[3]{2xy}}{y}$ e. $3\sqrt{5}-6$ f. $\dfrac{3\sqrt{6}+3\sqrt{2}+6+2\sqrt{3}}{4}$

3. $\dfrac{x-16}{x+6\sqrt{x}+8}$

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Mini Lecture 10.6
Radical Equations

Learning Objectives:

1. Solve radical equations.
2. Use models that are radical functions to solve problems.

Examples:

Solve.

1. a. $\sqrt{4y+1}=1$ b. $\sqrt{6x-4}=-2$ c. $\sqrt{3a+1}-3=1$

   d. $\sqrt{4x-3}-5=0$ e. $\sqrt{x+4}=\sqrt{2x-5}$ f. $\sqrt[3]{3x}+4=7$

2. a. $\sqrt{3y+1}=y-3$ b. $\sqrt[3]{2a+7}=-1$ c. $\sqrt{x+10}=x-2$

   d. $\sqrt[4]{6a+7}=\sqrt[4]{a+2}$ e. $y-1=\sqrt{6y+1}$ f. $\sqrt[3]{3x+5}=\sqrt[3]{5-2x}$

Solve. Each of the following examples will require squaring both sides twice.

3. a. $\sqrt{x-8}=\sqrt{x}-2$ b. $\sqrt{a+5}=\sqrt{a-3}+2$ c. $\sqrt{y+8}=\sqrt{y-4}+2$

Solve.

4. a. $(5x+7)^{\frac{1}{3}}=2$ b. $x+1=(5x+1)^{\frac{1}{2}}$ c. $(5x-1)^{\frac{1}{2}}-6=1$

Teaching Notes:

- When solving equations with radicals, isolate the radical on one side first.
- Raise both sides of the equation to the power that is the index of the radical in order to eliminate the radical. Sometimes this step must be done a second time to clear the equation of all radicals.
- Always check all solutions for extraneous solutions.

Answers: 1. a. 0 b. no solution c. 5 d. 7 e. 9 f. 9 2. a. 8 b. –4 c. 6 d. –1 e. 8 f. 0 3. a. 9 b. 4 c. 8 4. a. $\frac{1}{5}$ b. 0, 3 c. 10

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Mini Lecture 10.7
Complex Numbers

Learning Objectives:

1. Express square roots of negative numbers in terms of i.
2. Add and subtract complex numbers.
3. Multiply complex numbers.
4. Divide complex numbers.
5. Simplify powers of i.

Examples:

1. Write as a multiple of i.
   a. $\sqrt{-81}$ b. $\sqrt{-13}$ c. $\sqrt{-60}$

2. Perform the indicated operation. Write the result in the form $a + bi$.
   a. $(3+4i)+(4-7i)$
   b. $(1+i)-(3+4i)$
   c. $4i(4-6i)$
   d. $(3+2i)(5-3i)$
   e. $\sqrt{-3}\cdot\sqrt{-4}$

3. Divide and simplify the form $a+bi$.
   a. $\dfrac{4+3i}{2i}$ b. $\dfrac{1+i}{2-3i}$

4. Simplify.
   a. i^{21} b. i^{30} c. i^{40} d. i^{7}

Teaching Notes:

- The **imaginary unit** i is defined as $i=\sqrt{-1}$ where $i^2=-1$.
- If b is a positive real number, then $\sqrt{-b}=\sqrt{b(-1)}=\sqrt{b}\cdot\sqrt{-1}=i\sqrt{b}$.
- The set of all numbers in the form $a+bi$, a is the **real part**, b is called the **imaginary part** of the **complex number** $a+bi$.
- When adding or subtracting complex numbers, add or subtract their real parts. Then add or subtract their imaginary parts and express the answer as a complex number.
- Students need to be remained often to write their answers in $a + bi$ form.

Answers: 1. a. $9i$ b. $i\sqrt{13}$ c. $2i\sqrt{15}$ 2. a. $7-3i$ b. $-2-3i$ c. $24+16i$ d. $21+i$ e. $-2\sqrt{3}$ 3. a. $\dfrac{3-4i}{2}$ or $\dfrac{3}{2}-2i$ b. $\dfrac{-1+5i}{13}$ or $-\dfrac{1}{13}+\dfrac{5}{13}i$ 4. a. i b. -1 c. 1 d. $-i$

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