Chapter 11 Test

1. $$\begin{aligned} 2x^2 - 5 &= 0 \\ 2x^2 &= 5 \\ x^2 &= \frac{5}{2} \\ x &= \pm\sqrt{\frac{5}{2}} \end{aligned}$$

Rationalize the denominators.

$$x = \pm\frac{\sqrt{5}}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \pm\frac{\sqrt{10}}{2}$$

The solution set is $\left\{\pm\frac{\sqrt{10}}{2}\right\}$.

2. $$\begin{aligned} (x-3)^2 &= 20 \\ x - 3 &= \pm\sqrt{20} \\ x &= 3 \pm \sqrt{4 \cdot 5} \\ x &= 3 \pm 2\sqrt{5} \end{aligned}$$

The solution set is $\left\{3 \pm 2\sqrt{5}\right\}$.

3. $x^2 - 16x + \_\_\_\_\_$

Since $b = -16$, add $\left(\frac{b}{2}\right)^2 = \left(\frac{-16}{2}\right)^2 = (-8)^2 = 64$.

$x^2 - 16x + 64 = (x-8)^2$

4. $x^2 + \frac{2}{5}x + \_\_\_\_\_$

Since $b = \frac{2}{5}$, add $\left(\frac{1}{2}b\right)^2 = \left(\frac{1}{2}\cdot\frac{2}{5}\right)^2 = \left(\frac{1}{5}\right)^2 = \frac{1}{25}$.

$x^2 + \frac{2}{5}x + \frac{1}{25} = \left(x + \frac{1}{5}\right)^2$

5. $x^2 - 6x + 7 = 0$

$x^2 - 6x \quad = -7$

Since $b = -6$, add $\left(\frac{b}{2}\right)^2 = \left(\frac{-6}{2}\right)^2 = (-3)^2 = 9$.

$$x^2 - 6x + 9 = -7 + 9$$
$$(x-3)^2 = 2$$

Apply the square root property.

$$x - 3 = \pm\sqrt{2}$$
$$x = 3 \pm \sqrt{2}$$

The solution set is $\left\{3 \pm \sqrt{2}\right\}$.

6. Use the Pythagorean Theorem.

$$50^2 + 50^2 = x^2$$
$$2500 + 2500 = x^2$$
$$5000 = x^2$$
$$\pm\sqrt{5000} = x$$
$$\pm\sqrt{2500\cdot 2} = x$$
$$\pm 50\sqrt{2} = x$$

The solutions are $\pm 50\sqrt{2}$ feet. Disregard $-50\sqrt{2}$ feet because we can't have a negative length measurement. The width of the pond is $50\sqrt{2}$ feet.

7. $d = \sqrt{\left(2-(-1)\right)^2 + (-3-5)^2}$

$= \sqrt{(3)^2 + (-8)^2}$

$= \sqrt{9 + 64}$

$= \sqrt{73}$

≈ 8.54

8. midpoint $= \left(\frac{-5+12}{2}, \frac{-2+(-6)}{2}\right)$

$= \left(\frac{7}{2}, \frac{-8}{2}\right)$

$= \left(\frac{7}{2}, -4\right)$

9. $3x^2 + 4x - 2 = 0$

$a = 3 \quad b = 4 \quad c = -2$

Find the discriminant.

$b^2 - 4ac = 4^2 - 4(3)(-2)$

$= 16 + 24 = 40$

Since the discriminant is greater than zero but not a perfect square, there are two real irrational solutions.

10. $x^2 = 4x - 8$

$x^2 - 4x + 8 = 0$

$a = 1 \quad b = -4 \quad c = 8$

Find the discriminant.

$b^2 - 4ac = (-4)^2 - 4(1)(8)$

$= 16 - 32 = -16$

Since the discriminant is negative, there are two imaginary solutions which are complex conjugates.

11. $2x^2 + 9x = 5$

$2x^2 + 9x - 5 = 0$

$(2x-1)(x+5) = 0$

Apply the zero-product principle.

$2x - 1 = 0$ and $x + 5 = 0$

$2x = 1 \qquad x = -5$

$x = \frac{1}{2}$

The solution set is $\left\{-5, \frac{1}{2}\right\}$.

12. $x^2+8x+5=0$

Solve using the quadratic formula.

$a=1 \quad b=8 \quad c=5$

$$x=\frac{-8\pm\sqrt{8^2-4(1)(5)}}{2(1)}$$
$$=\frac{-8\pm\sqrt{64-20}}{2}$$
$$=\frac{-8\pm\sqrt{44}}{2}$$
$$=\frac{-8\pm\sqrt{4\cdot 11}}{2}$$
$$=\frac{-8\pm 2\sqrt{11}}{2}$$
$$=\frac{2\left(-4\pm\sqrt{11}\right)}{2}=-4\pm\sqrt{11}$$

The solution set is $\left\{-4\pm\sqrt{11}\right\}$.

13. $(x+2)^2+25=0$

$(x+2)^2=-25$

Apply the square root principle.

$x+2=\pm\sqrt{-25}$

$x=-2\pm 5i$

The solution set is $\{-2\pm 5i\}$.

14. $2x^2-6x+5=0$

$a=2 \quad b=-6 \quad c=5$

$$x=\frac{-(-6)\pm\sqrt{(-6)^2-4(2)(5)}}{2(2)}$$
$$=\frac{6\pm\sqrt{36-40}}{4}$$
$$=\frac{6\pm\sqrt{-4}}{4}$$
$$=\frac{6\pm 2i}{4}=\frac{6}{4}\pm\frac{2}{4}i=\frac{3}{2}\pm\frac{1}{2}i$$

The solution set is $\left\{\frac{3}{2}\pm\frac{1}{2}i\right\}$.

15. Because the solution set is $\{-3,7\}$, we have

$x=-3 \quad \text{or} \quad x=7$

$x+3=0 \qquad x-7=0$

Apply the zero-product principle in reverse.

$(x+3)(x-7)=0$

$x^2-7x+3x-21=0$

$x^2-4x-21=0$

16. Because the solution set is $\{-10i,10i\}$, we have

$x=-10i \quad \text{or} \quad x=10i$

$x+10i=0 \qquad x-10i=0$

Apply the zero-product principle in reverse.

$(x+10i)(x-10i)=0$

$x^2-100i^2=0$

$x^2-100(-1)=0$

$x^2+100=0$

17. **a.** 2014 is 11 years after 2003.

$f(x)=2.4x^2+0.7x+29$

$f(11)=2.4(11)^2+0.7(11)+29$

$=327.1$

≈ 327

According to the function, in 2014 there were 327 “Bicycle Friendly” communities. This underestimates the number shown in the graph by 5.

b. $f(x)=2.4x^2+0.7x+29$

$1206=2.4x^2+0.7x+29$

$0=2.4x^2+0.7x-1177$

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$
$$x=\frac{-(0.7)\pm\sqrt{(0.7)^2-4(2.4)(-1177)}}{2(2.4)}$$
$$=\frac{-0.7\pm\sqrt{0.49+11299.2}}{2(2.4)}$$
$$=\frac{-0.7\pm\sqrt{11299.69}}{4.8}$$
$$=\frac{-0.7\pm 106.3}{4.8}$$

$x=22 \quad \text{or} \quad -22.3$

According to the function, there will be 1206 “Bicycle Friendly” communities 22 years after 2003, or 2025.

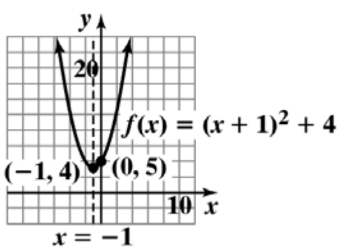
18. $f(x) = (x+1)^2 + 4$

Since $a = 1$ is positive, the parabola opens upward. The vertex of the parabola is $(h,k) = (-1,4)$ and the axis of symmetry is $x = -1$. Replace $f(x)$ with 0 to find x–intercepts.

$$0 = (x+1)^2 + 4$$
$$-4 = (x+1)^2$$

This will be result in complex solutions. As a result, there are no
x–intercepts. Set $x = 0$ and solve for y to obtain the y–intercept.

$$y = (0+1)^2 + 4 = 1 + 4 = 5$$



Axis of symmetry: $x = -1$.

19. $f(x) = x^2 - 2x - 3$

Since $a = 1$ is positive, the parabola opens upward. The x–coordinate of the vertex of the parabola is

$$-\frac{b}{2a} = -\frac{-2}{2(1)} = -\frac{-2}{2} = 1$$ and the

y–coordinate of the vertex of the parabola is

$$\begin{aligned} f\left(-\frac{b}{2a}\right) &= f(1) \\ &= 1^2 - 2(1) - 3 \\ &= 1 - 2 - 3 \\ &= -4. \end{aligned}$$

The vertex is $(1,-4)$. Replace $f(x)$ with 0 to find x–intercepts.

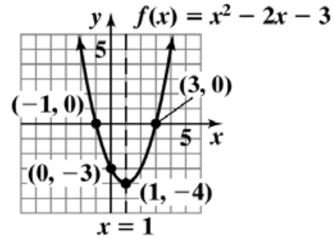
$$0 = x^2 - 2x - 3$$
$$0 = (x-3)(x+1)$$

Apply the zero-product principle.

$$x - 3 = 0 \quad \text{or} \quad x + 1 = 0$$
$$x = 3 \qquad\qquad x = -1$$

The x–intercepts are –1 and 3. Set $x = 0$ and solve for y to obtain the y–intercept.

$$y = 0^2 - 2(0) - 3 = -3$$



Axis of symmetry: $x = 1$.

20. $s(t) = -16t^2 + 64t + 5$

Since $a = -16$ is negative, the function opens downward and has a maximum at

$$x = -\frac{b}{2a} = -\frac{64}{2(-16)} = -\frac{64}{-32} = 2.$$

The ball reaches its maximum height in two seconds. The maximum height is

$$\begin{aligned} s(2) &= -16(2)^2 + 64(2) + 5 \\ &= -16(4) + 128 + 5 \\ &= -64 + 128 + 5 = 69. \end{aligned}$$

The baseball reaches a maximum height of 69 feet after 2 seconds.

21. $0 = -16t^2 + 64t + 5$

Solve using the quadratic formula.
$a = -16 \quad b = 64 \quad c = 5$

$$\begin{aligned} x &= \frac{-64 \pm \sqrt{64^2 - 4(-16)(5)}}{2(-16)} \\ &= \frac{-64 \pm \sqrt{4096 + 320}}{-32} \\ &= \frac{-64 \pm \sqrt{4416}}{-32} \\ &\approx 4.1 \text{ or } -0.1 \end{aligned}$$

Disregard –0.1 since we cannot have a negative time measurement. The solution is 4.1 and we conclude that the baseball hits the ground in approximately 4.1 seconds.

22. $f(x) = -x^2 + 46x - 360$

Since $a = -1$ is negative, the function opens downward and has a maximum at $x = -\frac{b}{2a} = -\frac{46}{2(-1)} = -\frac{46}{-2} = 23.$

$f(23) = -23^2 + 46(23) - 360 = 169$

Profit is maximized when 23 computers are manufactured. This produces a profit of \$169 hundreds or \$16,900.

23. Let $u = 2x - 5.$

$$(2x-5)^2 + 4(2x-5) + 3 = 0$$
$$u^2 + 4u + 3 = 0$$
$$(u+3)(u+1) = 0$$
$$u+3 = 0 \quad \text{or} \quad u+1 = 0$$
$$u = -3 \qquad\qquad u = -1$$

Replace u by $2x - 5.$

$$2x-5 = -3 \quad \text{or} \quad 2x-5 = -1$$
$$2x = 2 \qquad\qquad 2x = 4$$
$$x = 1 \qquad\qquad x = 2$$

The solution set is $\{1, 2\}$.

24. Let $u = x^2.$

$$x^4 - 13x^2 + 36 = 0$$
$$\left(x^2\right)^2 - 13x^2 + 36 = 0$$
$$u^2 - 13u + 36 = 0$$
$$(u-9)(u-4) = 0$$
$$u-9 = 0 \quad \text{or} \quad u-4 = 0$$
$$u = 9 \qquad\qquad u = 4$$

Replace u by $x^2.$

$$x^2 = 9 \quad \text{or} \quad x^2 = 4$$
$$x = \pm 3 \qquad\qquad x = \pm 2$$

The solution set is $\{-3, -2, 2, 3\}$.

25. Let $u = x^{1/3}$.

$$x^{2/3} - 9x^{1/3} + 8 = 0$$
$$\left(x^{1/3}\right)^2 - 9x^{1/3} + 8 = 0$$
$$u^2 - 9u + 8 = 0$$
$$(u-8)(u-1) = 0$$
$$u - 8 = 0 \quad \text{or} \quad u - 1 = 0$$
$$u = 8 \qquad\qquad u = 1$$

Replace u by $x^{1/3}$.

$$x^{1/3} = 8 \quad \text{or} \quad x^{1/3} = 1$$
$$x = 8^3 = 512 \qquad x = 1^3 = 1$$

The solution set is $\{1, 512\}$.