Mini Lecture 11.1
The Square Root Property and Completing the Square; Distance and Midpoint Formulas

Learning Objectives:

1. Solve quadratic equations using the square root property.
2. Complete the square of a binomial.
3. Solve quadratic equations by completing the square.
4. Solve problems using the square root property.
5. Find the distance between two points.
6. Find the midpoint of a line segment.

Examples:

1. Solve using the square root property.
   a. $3x^2 = 18$ b. $5x^2 - 7 = 0$
   c. $16x^2 + 25 = 0$ d. $(x-2)^2 = 3$

2. What term should be added to each binomial so that it becomes a perfect square trinomial? Write and factor the trinomial.
   a. $x^2 + 8x$ b. $x^2 - 5x$ c. $x^2 + \frac{1}{2}x$

3. Solve by completing the square.
   a. $x^2 + 10x + 12 = 0$ b. $2x^2 - 6x - 5 = 0$

4. Use the compound interest formula to find the annual interest rate, r.
   a. In 2 years, an investment of \$3000 grows to \$3307.50.
   b. In 4 years, an investment of \$6000 grows to \$8784.60.

5. Find the distance between each pair of points. If necessary, round answers to two decimals places.
   a. (6, 4) and (–2, 2) b. (1, 4) and (–3, 5)

6. Find the midpoint of the line segment with the given endpoints.
   a. (3, 7) and (9, 5) b. (3, –7) and (–4, –2)

Teaching Notes:

- The **square root property** states if u is an algebraic express and d is a non-zero real number, then $u^2 = d$ has two solutions. If $u^2 = d$, then $u = \sqrt{d}$ or $u = -\sqrt{d}$.
- When **completing the square**, if $x^2 + bx$ is a binomial, then by adding the square of half the coefficient of x, $\left(\frac{b}{2}\right)^2$, a perfect square trinomial will result

  $$x^2 + bx + \left(\frac{b}{2}\right)^2 = (x + \frac{b}{x})^2.$$

- If solving quadratic equations by completing the square, be sure when you add a constant term to one side of the equation to complete the square, be certain to add the same constant to the other side of the equation.
- The formula for compound interest is $A = P(1+r)^t$ where A is the account balance, t is the years, P is the principal originally invested, and r is the interest rate.
- The distance, d, between the points (x, y) and $(x_2,\ y_2)$, the coordinates of the segment's midpoints are: $\left(\frac{x_1 + x_2}{2},\ \frac{y_1 + y_2}{2}\right)$

Answers: 1. a. $\pm\sqrt{6}$ b. $\pm\frac{\sqrt{35}}{5}$ c. $\pm\frac{5}{4}i$ d. $2\pm\sqrt{3}$ 2. a. $x^2 + 8x + 16 = (x+4)^2$

b. $x^2 - 5x + \frac{25}{4} = (x - \frac{5}{2})^2$ c. $x^2 + \frac{1}{2}x + \frac{1}{16} = (x + \frac{1}{4})^2$ 3. a. $-5\pm\sqrt{13}$ b. $\frac{3\pm\sqrt{19}}{2}$

4. a. 5% b. 10% 5.a. 10 units b. $\sqrt{17}$ or 4.12 units 6.a. (6, 6) b. $\left(-\frac{1}{2},\ -\frac{9}{2}\right)$

Mini Lecture 11.2
The Quadratic Formula

Learning Objectives:

1. Solve quadratic equations using the quadratic formula.
2. Use the discriminant to determine the number and type of solutions.
3. Determine the most efficient method to use when solving a quadratic equations.
4. Write quadratic equations from solutions.
5. Use the quadratic formula to solve problems.

Examples:

1. Solve using the quadratic equation.

   a. $2x^2 - 5x = 3$ b. $3x^2 - 2x + 3 = 0$ c. $x^2 - 6x + 3 = 0$

   d. $3x^2 + 16x = -5$ e. $12x^2 - 4x + 5 = 0$ f. $x^2 + 6x = -13$

- Which problems in #1 could have been solved by factoring?

2. Find the value of the discriminant. Tell what kind and how many solutions each equation would have if solved.

   a. $9x^2 - 6x + 1 = 0$ b. $5x^2 = 4x - 6$ c. $x^2 + 4x = 1$

   d. $3x^2 - 4x = 2 = 0$ e. $2x^2 - 2x = 24$ f.. $4x^2 - 1 = 0$

3. Write an equation with the given solution set.

   a. $\{4, -2\}$ b. $\left\{\frac{-2}{3}, \frac{1}{5}\right\}$ c. $\{-\sqrt{5}, \sqrt{5}\}$ d. $\{3i, -3i\}$

Teaching Notes:

- Students must memorize the quadratic formula.
- The quadratic formula can always be used to solve a quadratic equation.
- Students need to be reminded of the standard form of a quadratic equation, $ax^2 + bx + c = 0$. When using the quadratic formula the equation should be in standard form.
- Have students write the quadratic formula each time they work a problem.
- Students need to have a firm grasp on how to choose a method for solving equations.

Answers: 1. a. $\left\{\frac{-1}{2}, 3\right\}$ b. $\left\{\frac{1 \pm 2i\sqrt{2}}{3}\right\}$ c. $\{3 \pm \sqrt{6}\}$ d. $\left\{\frac{-1}{3}, 5\right\}$ e. $\{-3 \pm 2i\}$ f. $\left\{\frac{2 \pm \sqrt{10}}{2}\right\}$ 2. a. $D = 0$; one rational solution b. $D = -104$; two imaginary solutions c. $D = 20$: two irrational solutions d. $D = -8$; two imaginary solutions e. $D = 196$; two rational solutions f. $D = 16$; two rational solutions

Mini Lecture 11.3

Quadratic Functions and Their Graphs

Learning Objectives:

1. Recognize characteristics of parabolas.
2. Graph parabolas in the form $f(x) = a(x-h)^2 + k$.
3. Graph parabolas in the form $f(x) = ax^2 + bx + c$.
4. Determine a quadratic function's minimum or maximum value.
5. Solve problems involving a function's minimum of maximum value.

Examples:

1. Graph the quadratic function. Identify the vertex, x and y intercepts. Use the graph to identify the function's domain and its range.

   a. $f(x) = -(x+2)^2 - 3$

   b. $f(x) = x^2 - 2x + 2$

   c. $f(x) = (x+1)^2 + 4$

   d. $f(x) = x^2 - 4$

Teaching Notes:

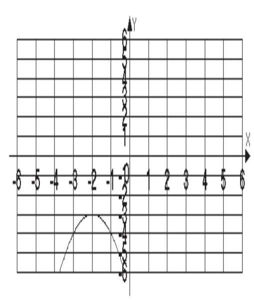
- The graph of a quadratic function $f(x) = ax^2 + bx + c,\ a \neq 0$ is called a parabola.
- Parabolas are symmetric with respect to an imaginary line which is called the **axis of symmetry**.
- To graph quadratic functions with equations in the form $f(x) = a(x-h)^2 + k$
  - * Determine if the parabola opens up ($a > 0$) or opens down ($a < 0$).
  - * Determine the vertex of the parabola (h, k).
  - * Determine the x-intercept by replacing $f(x)$ with 0 and solving.
  - * Determine the y-intercept by replacing x with 0 and solving.
- The vertex of a parabola whose equation is $f(x) = ax^2 + bx + c$ is $\left(\frac{-b}{2a},\ f\left(\frac{-b}{2a}\right)\right)$.
- With a quadratic function, a maximum or minimum value will occur at the vertex point

  * if $a > 0$, then the minimum occurs at $x = \frac{-b}{2a}$ and the minimum value is $f\left(\frac{-b}{2a}\right)$.

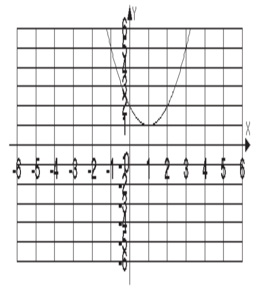
  * if $a < 0$, then the maximum occurs at $x = \frac{-b}{2a}$ and the maximum value is $f\left(\frac{-b}{2a}\right)$.

Answers:

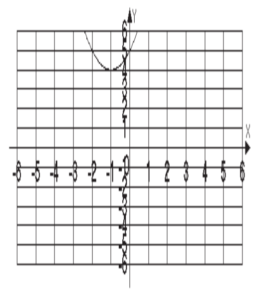
1. a. vertex (–2, –3)
   x-intercept – none
   y-intercept – (0, –7)
   domain $(-\infty, \infty)$
   range $(-\infty, -3]$



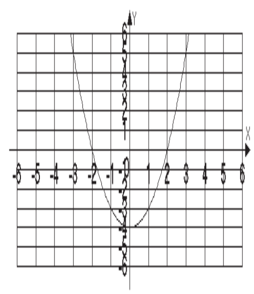
1. b. vertex (1, 1)
x-intercept – none
y-intercept – (0, 2)
domain $\left(-\infty, \infty\right)$
range $\left[1, \infty\right)$



1. c. vertex (–1, 4)
x-intercept – (–3, 0)(1
y-intercept – (0, 3)
domain $\left(-\infty, \infty\right)$
range $\left(-\infty, 4\right]$



1. d. vertex (0, 4)
x-intercept – (–2, 0)(2, 0)
y-intercept – (0, –4)
domain $\left(-\infty, \infty\right)$
range $\left[-4, \infty\right)$



Mini Lecture 11.4
Equations Quadratic in Form

Learning Objectives:

1. Solve equations that are quadratic in form.

Examples:

Solve.

1. a. $x^4 - x^2 = 12$ b. $y^4 + 15 = 8y^2$ c. $4x^4 - 7x^2 - 2 = 0$

2. a. $(y+3)^2 - 3(y+3) = 70$ b. $(x-2)^2 - 4(x-2) - 60 = 0$ c. $3(4a-1)^2 + (4a-1) = 10$

3. a. $x - \sqrt{x} - 2 = 0$ b. $3y + \sqrt{y} = 2$ c. $x - 6\sqrt{x} + 8 = 0$

4. a. $y^{\frac{2}{3}} - 2y^{\frac{1}{3}} - 8 = 0$ b. $x^{-2} - 8x^{-1} + 7 = 0$ c. $y^{\frac{2}{3}} - 5y^{\frac{1}{3}} + 6 = 0$

Teaching Notes:

- An equation that is "quadratic in form" can be rewritten as a quadratic equation using an appropriate substitution.
- Remind students that if both sides of an equation are raised to an even power, they must check the solutions for extraneous solutions.
- A fourth degree equation will have four solutions.

Answers: 1. a. $\pm 2, \pm 3i$ b. $\pm\sqrt{5}, \pm\sqrt{3}$ c. $\pm\frac{1}{2}i, \pm\sqrt{2}$ 2. a. 7, –10 b. 12, –4 c. $\frac{2}{3}, -\frac{1}{4}$
3. a. 4 b. $\frac{4}{9}$ c. 16, 4 4. a. 64, –8 b. 1, $\frac{1}{7}$ c. 8, 27