Mini Lecture 12.1
Exponential Functions

Learning Objectives:

1. Evaluate exponential functions.
2. Graph exponential functions.
3. Evaluate functions with base *e*.
4. Use compound interest formulas.

Examples:

1. Approximate each number using a calculator, round your answer to two decimal places.
   a. $2^{1.2}$ b. $4^{\sqrt{2}}$ c. $e^{1.1}$ d. $e^{-0.25}$

2. Graph each function by making a table of coordinates.
   a. $f(x) = 3^x$ b. $g(x) = \left(\frac{2}{3}\right)^x$

3. Graph *f* and *g* in the same rectangular coordinate system. Select integers from –2 to 2 for *x*. Then describe how the graph of *g* is related to the graph of *f*.
   a. $f(x) = 4^x$ and $g(x) = 4^{x+1}$
   b. $f(x) = 3^x$ and $g(x) = 3^x + 2$

4. Find an accumulated value of an investment of $7000 for 5 years at an interest rate of 4% if the money is:
   a. compounded monthly.
   b. compounded semi-annually.
   c. compounded continuously.
   Use the compound interest formulas to solve. Round answers to the nearest cent.

Teaching Notes:

- The **exponential function** *f* with base *b* is $f(x) = b^x$ or $y = b^x$, $b > 0$ and $b \neq 1$ and *x* is any real number.
- The **domain** of an exponential function consists of all real numbers.
- The **range** of an exponential function consists of all positive real numbers.
- If $b > 1$, the graph of the exponential function will go up to the right and is an increasing function. The larger the value of *b*, the steeper the increase.
- If $0 < b < 1$, the graph of the exponential function goes down to the right and is a decreasing function. The small the value of *b*, the steeper the decrease.
- An irrational number, *e*, often appears as a base in applied exponential functions.
- The number *e* is called the **natural base**. "e"$\approx 2.71828...$

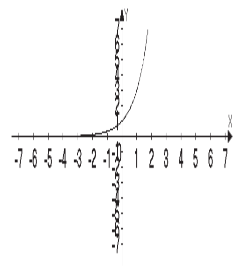
- The function $f(x) = e$ is called the **natural exponential function**.
- Formulas for **compound interest**:

  * for n compounding per year $A = P(1 + \frac{r}{n})^{nt}$

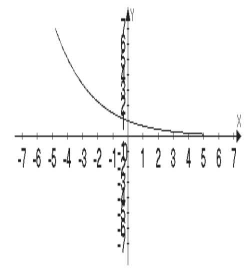
  * for continuous compounding $A = Pe^{rt}$

  After t years, the balance A in an account with principal P and as annual interest rate r.

<u>Answers</u>: 1. a. 2.30 b. 7.10 c. 3.00 d. 0.78 2. a.



b.



3. a. The graph of g is the graph of f shifted 1 unit to the left b. The graph of g is the graph of f shifted up 2 units 4. a. $8546.98 b. $8532.96 c. $8549.82

Mini Lecture 12.2
Logarithmic Functions

Learning Objectives:

1. Change from logarithmic to exponential form.
2. Change from exponential to logarithmic form.
3. Evaluate logarithms.
4. Use basic logarithmic properties.
5. Graph logarithmic functions.
6. Find the domain of a logarithmic function.
7. Use common logarithms.
8. Use natural logarithms.

Examples:

1. Write each equation in its equivalent exponential form.
   a. $5 = \log_2 x$ b. $2 = \log_b 16$ c. $\log_5 18 = y$

2. Write each equation in its equivalent logarithmic form.
   a. $3^4 = 81$ b. $b^3 = 8$ c. $e^y = 11$

3. Evaluate.
   a. $\log_{10} 10000$ b. $\log_5 5$ c. $\log_{125} 5$
   d. $\log_4 4$ e. $\log_7 1$ f. $\log_3 3^4$
   g. $5^{\log_5 12}$ h. $\ln 1$ i. $\ln e$

4. Graph $f(x) = 4^x$ and $g(x) = \log_4 x$ in the same rectangular coordinate system.

5. Find the domain of each function:
   a. $f(x) = \log_2 (x-3)$ b. $g(x) = \ln(5-x)$

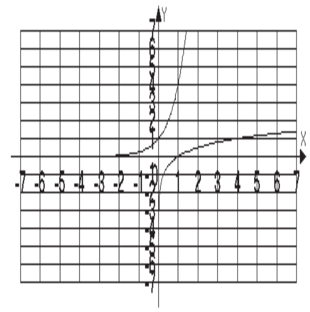
Teaching Notes:

- The function $f(x) = \log_b x$ is the **logarithmic function with base *b***.
- The logarithmic form $y = \log_b x$ is equivalent to the exponential form $b^y = x$.
- The **domain** of a logarithmic function of the form $f(x) = \log_b x$ is the set of all positive real numbers.
- The **domain** of $f(x) = \log_b [g(x)]$ consists of all x for which $g(x) > 0$.
- The logarithmic function with base 10 is called the **common logarithmic function**.
- Properties of Common Logarithms

General Properties	Common Logarithms	Natural Logarithms
$\log_b 1 = 0$	$\log 1 = 0$	$\ln 1 = 0$
$\log_b b = 1$	$\log 10 = 1$	$\ln e = 1$
$\log_b b^x = x$	$\log 10^x = x$	$\ln e^x = x$
$b^{\log_b x} = x$	$10^{\log x} = x$	$e^{\ln x} = x$

Answers: 1. a. $2^5 = x$ b. $b^2 = 16$ c. $5^y = 18$ 2. a. $4 = \log_3 81$ b. $3 = \log_b 8$ c. $y = \log_e 11$

3. a. 4 b. 1 c. $\frac{1}{3}$ d. 1 e. 0 f. 4 g. 12 h. 0 i. 1 4.



5. a. $(3, \infty)$ b. $(-\infty, 5)$

Mini Lecture 12.3
Properties of Logarithms

Learning Objectives:
1. Use the product rule.
2. Use the quotient rule.
3. Use the power rule.
4. Expand logarithmic expressions.
5. Condense logarithmic expressions.
6. Use the change-of-base property.

Examples:
Expand.

1. a. $\log_3(4\cdot 18)$ b. $\log_8(12\cdot 8)$ c. $\log_5(5x)$

2. a. $\log_3\left(\frac{3}{x}\right)$ b. $\ln\left(\frac{e^3}{5}\right)$ c. $\log\left(\frac{10}{x}\right)$

3. a. $\log_4 7^3$ b. $\log_2 y^3 z$ c. $\log_5 3^{\frac{1}{2}}$

4. a. $\log\left(\frac{x^2}{\sqrt[3]{y}}\right)$ b. $\log_5 \frac{x}{y^4}$ c. $\log_7\left(\frac{6y^2}{\sqrt[4]{x}}\right)$

Condense. Write as a single logarithm.

5. a. $\log_8 5 + \log_8 x$ b. $\log_6 18 + \log_6 2 - \log_6 9$
c. $2\log_3 5 + \log_3 2$ d. $\log_2 x + \log_2(x-3) - \log_2 3$

Use the change-of-base property and your calculator to find the decimal approximation.

6. a. $\log_6 14$ b. $\log_9 27$ c. $\log_7 15$

Teaching Notes:
- Students must <u>know</u> properties and rules of logarithms and since this will be new to most students, a lot of practice is recommended.
- As rules are introduced, show several examples with numbers instead of letters.
- Make sure students know how to use their calculators to find decimal approximations.

<u>Answers</u>: 1. a. $\log_4 + \log_3 18$ b. $\log_8 12 + 1$ c. $1 + \log_5 x$ 2. a. $1 - \log_3 x$ b. $3 - \ln 5$ c. $1 - \log x$ 3. a. $3\log_4 7$ b. $3\log_2 y + \log_2 z$ c. $\frac{1}{2}\log_5 3$ 4. a. $2\log x - \frac{1}{3}\log y$ b. $\log_5 x - 4\log_5 y$ c. $\log_7 6 + 2\log_7 y - \frac{1}{4}\log_7 x$ 5. a. $\log_8 5x$ b. $\log_6 4$ c. $\log_3 50$ d. $\log_2\left(\frac{x^2-3x}{3}\right)$ 6. a. 1.4729 b. 1.5 c. 1.3917

Mini Lecture 12.4
Exponential and Logarithmic Equations

Learning Objectives:

1. Use like bases to solve exponential equations.
2. Use logarithms to solve exponential equations.
3. Use exponential form to solve logarithmic equations.
4. Use the one-to-one property of logarithms to solve logarithmic equations.
5. Solve applied problems involving exponential and logarithmic equations.

Examples:

1. Solve

   a. $3^x = \frac{1}{81}$ b. $8^x = 16$ c. $27^x = 9$

2. Find the solution set and then use a calculator to obtain a decimal approximation to two decimal places for the solution.

   a. $3^x = 120$ b. $5^{3x} = 30$

3. Solve.

   a. $\log_4(x-3) = 2$ b. $\log_2 x + \log_2(x-2) = 3$

   c. $\log_4(x+1) - \log_4(x-2) = 1$ d. $5\ln 2x = 15$

Teaching Notes:

- An **exponential equation** is an equation containing a variable with an exponent.
- To solve exponential equations, express each side of the equal as a power of the same base and then set the exponents equal to each other. If $b^m = b^n$, then $m = n$.
- When using rational logarithms to solve exponential equations, first, isolate the exponential expressions. Next, take the natural logarithm on both sides of the equation, simplify and solve for the variable.
- A **logarithmic equation** is an equation containing a variable in a logarithmic expression.
- ALWAYS check proposed solutions of a logarithmic equation in the original equation. Exclude from the solution set any proposed solution that produces the logarithm of a negative number or the logarithm of 0.

Answers: 1. a. –4 b. $\frac{4}{3}$ c. $\frac{2}{3}$ 2. a. $\frac{\ln 120}{\ln 3} \approx 4.36$ b. $\frac{\ln 6}{3} = \ln 2 \approx 0.69$ 3. a. 19 b. 4 c. 3 d. $\frac{e^3}{2}$