78. Number of Seriously Mentally Ill Adults in the United States

Year	Number of Seriously Mentally Ill Adults
2006	9.0
2008	9.2
2010	9.4
2012	9.6

Source: U.S. Census Bureau

79. Percentage of Moderate Alcohol Users in the United States (Not Binge or Heavy Drinkers)

Age	Percentage of Moderate Drinkers
20	15%
25	24%
30	28%
35	32%
40	34%
45	35%

Source: Substance Abuse and Mental Health Services Administration

80. U.S. Electric Car Sales

Year	Number of Cars Sold
2010	5000
2011	19,000
2012	53,000
2013	98,000

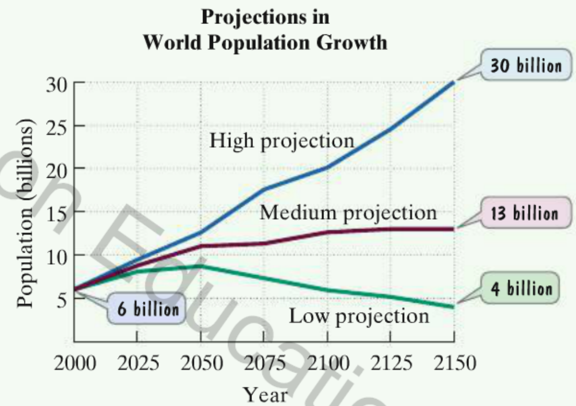
Source: Electric Drive Transportation Association

In Exercises 81–82, rewrite the equation in terms of base e. Express the answer in terms of a natural logarithm and then round to three decimal places.

81. $y = 73(2.6)^x$

82. $y = 6.5(0.43)^x$

83. The figure shows world population projections through the year 2150. The data are from the United Nations Family Planning Program and are based on optimistic or pessimistic expectations for successful control of human population growth. Suppose that you are interested in modeling these data using exponential, logarithmic, linear, and quadratic functions. Which function would you use to model each of the projections? Explain your choices. For the choice corresponding to a quadratic model, would your formula involve one with a positive or negative leading coefficient? Explain.



CHAPTER 12 TEST

Step-by-step test solutions are found on the Chapter Test Prep Videos available in MyMathLab® or on YouTube (search "BlitzerCombo5e" and click on "Channels").

1. Graph $f(x) = 2^x$ and $g(x) = 2^{x+1}$ in the same rectangular coordinate system.

2. Use $A = P\left(1 + \frac{r}{n}\right)^{nt}$ and $A = Pe^{rt}$ to solve this problem. Suppose you have \$3000 to invest. Which investment yields the greater return over 10 years: 6.5% compounded semiannually or 6% compounded continuously? How much more (to the nearest dollar) is yielded by the better investment?

3. Write in exponential form: $\log_5 125 = 3$.

4. Write in logarithmic form: $\sqrt{36} = 6$.

5. Graph $f(x) = 3^x$ and $g(x) = \log_3 x$ in the same rectangular coordinate system. Use the graphs to determine each function's domain and range.

In Exercises 6–8, simplify each expression.

6. $\ln e^{5x}$ **7.** $\log_b b$ **8.** $\log_6 1$

9. Find the domain: $f(x) = \log_5 (x - 7)$.

10. On the decibel scale, the loudness of a sound, in decibels, is given by $D = 10 \log \frac{I}{I_0}$, where I is the intensity of the sound, in watts per meter^2, and I_0 is the intensity of a sound barely audible to the human ear. If the intensity of a sound is $10^{12} I_0$, what is its loudness in decibels? (Such a sound is potentially damaging to the ear.)

In Exercises 11–12, use properties of logarithms to expand each logarithmic expression as much as possible. Where possible, evaluate logarithmic expressions without using a calculator.

11. $\log_4(64x^5)$

12. $\log_3\left(\frac{\sqrt[3]{x}}{81}\right)$

In Exercises 13–14, write each expression as a single logarithm.

13. $6 \log x + 2 \log y$

14. $\ln 7 - 3 \ln x$

15. Use a calculator to evaluate $\log_{15} 71$ to four decimal places.

In Exercises 16–23, solve each equation.

16. $3^{x-2} = 81$

17. $5^x = 1.4$

18. $400e^{0.005x} = 1600$

x	y
0	3
1	1
2	−1
3	−3
4	−5

	y
$\frac{1}{3}$	−1
1	0
3	1
9	2
27	3

	y
0	1
1	5
2	25
3	125
4	625

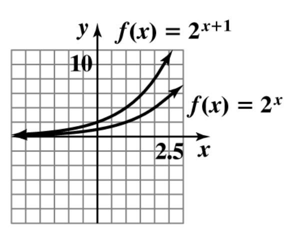
	y
0	12
1	3
2	0
3	3
4	12

solutions

Chapter 12 Test

1. $f(x) = 2^x$

$g(x) = 2^{x+1}$



2. Semiannual Compounding:

$$A = 3000\left(1+\frac{0.065}{2}\right)^{2(10)}$$
$$= 3000(1.0325)^{20} \approx 5687.51$$

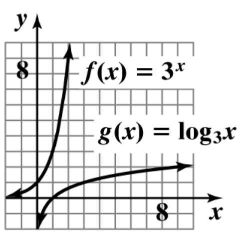
Continuous Compounding:
$A = 3000e^{0.06(10)} = 3000e^{0.6} \approx 5466.36$

Semiannual compounding at 6.5% yields a greater return. The difference in the yields is \$221.

3. $\log_5 125 = 3$
$$5^3 = 125$$

4. $\sqrt{36} = 6$
$$36^{\frac{1}{2}} = 6$$
$$\log_{36} 6 = \frac{1}{2}$$

5. $f(x) = 3^x$
$g(x) = \log_3 x$



Domain of *f*: $(-\infty, \infty)$.
Range of *f*: $(0, \infty)$

Domain of *g*: $(0, \infty)$
Range of *g*: $(-\infty, \infty)$.

6. Since $\ln e^x = x$, $\ln e^{5x} = 5x$.

7. $\log_b b = 1$ because $b^1 = b$.

8. $\log_6 1 = 0$ because $6^0 = 1$.

9. $f(x) = \log_5(x-7)$
$$x - 7 > 0$$
$$x > 7$$

The domain of *f* is $(7, \infty)$.

10. $D = 10\log\frac{I}{I_0}$
$$D = 10\log\frac{10^{12} I_0}{I_0}$$
$$= 10\log 10^{12} = 10(12) = 120$$

The sound has a loudness of 120 decibels.

11. $\log_4(64x^5) = \log_4 64 + \log_4 x^5$
$$= 3 + 5\log_4 x$$

12. $\log_3 \frac{\sqrt[3]{x}}{81} = \log_3 \sqrt[3]{x} - \log_3 81$
$$= \log_3 x^{\frac{1}{3}} - 4 = \frac{1}{3}\log_3 x - 4$$

13. $6\log x + 2\log y = \log x^6 + \log y^2$
$$= \log(x^6 y^2)$$

14. $\ln 7 - 3\ln x = \ln 7 - \ln x^3 = \ln\left(\frac{7}{x^3}\right)$

15. $\log_{15} 71 = \frac{\ln 71}{\ln 15} \approx 1.5741$

16. $3^{x-2} = 81$
$$3^{x-2} = 3^4$$
$$x - 2 = 4$$
$$x = 6$$

The solution set is $\{6\}$.

17. $5^x = 1.4$
$$\ln 5^x = \ln 1.4$$
$$x\ln 5 = \ln 1.4$$
$$x = \frac{\ln 1.4}{\ln 5} \approx 0.21$$

The solution set is $\left\{\frac{\ln 1.4}{\ln 5} \approx 0.21\right\}$.

18. $400e^{0.005x} = 1600$

$$e^{0.005x} = \frac{1600}{400}$$

$$\ln e^{0.005x} = \ln 4$$
$$0.005x = \ln 4$$
$$x = \frac{\ln 4}{0.005} \approx 277.26$$

The solution set is $\left\{\frac{\ln 4}{0.005} \approx 277.26\right\}$.