Review Exercises

169. Solve: $8 - 6x > 4x - 12$. (Section 2.7, Example 7)

170. Simplify: $24 \div 8 \cdot 3 + 28 \div (-7)$. (Section 1.8, Example 8)

171. List the whole numbers in this set:

$$\left\{-4, -\frac{1}{5}, 0, \pi, \sqrt{16}, \sqrt{17}\right\}.$$

(Section 1.3, Example 5)

Preview Exercises

Exercises 172–174 will help you prepare for the material covered in the first section of the next chapter. In each exercise, find the product.

172. $4x^3(4x^2 - 3x + 1)$

173. $9xy(3xy^2 - y + 9)$

174. $(x + 3)(x^2 + 5)$

GROUP PROJECT

CHAPTER 5

A large number can be put into perspective by comparing it with another number. For example, we put the $12.3 trillion national debt (Example 10) into perspective by comparing this number to the number of U.S. citizens. In Exercises 139–142, we put the $1.35 trillion budget deficit into perspective by comparing 1.35 trillion to the number of U.S. citizens, the distance around the world, the number of seconds in a year, and the height of the Washington Monument.

For this project, each group member should consult an almanac, a newspaper, or the Internet to find a number greater than one million. Explain to other members of the group the context in which the large number is used. Express the number in scientific notation. Then put the number into perspective by comparing it with another number.

Chapter 5 Summary

Definitions and Concepts	Examples
Section 5.1 Adding and Subtracting Polynomials	
A polynomial is a single term or the sum of two or more terms containing variables with whole number exponents. A monomial is a polynomial with exactly one term; a binomial has exactly two terms; a trinomial has exactly three terms. The degree of a polynomial is the highest power of all the terms. The standard form of a polynomial is written in descending powers of the variable.	Polynomials Monomial: $2x^5$ (Degree is 5.) Binomial: $6x^3 + 5x$ (Degree is 3.) Trinomial: $7x + 4x^2 - 5$ (Degree is 2.)
To add polynomials, add like terms.	$(6x^3 + 5x^2 - 7x) + (-9x^3 + x^2 + 6x)$ $= (6x^3 - 9x^3) + (5x^2 + x^2) + (-7x + 6x)$ $= -3x^3 + 6x^2 - x$
The opposite, or additive inverse, of a polynomial is that polynomial with the sign of every coefficient changed. To subtract two polynomials, add the first polynomial and the opposite of the polynomial being subtracted.	$(5y^3 - 9y^2 - 4) - (3y^3 - 12y^2 - 5)$ $= (5y^3 - 9y^2 - 4) + (-3y^3 + 12y^2 + 5)$ $= (5y^3 - 3y^3) + (-9y^2 + 12y^2) + (-4 + 5)$ $= 2y^3 + 3y^2 + 1$

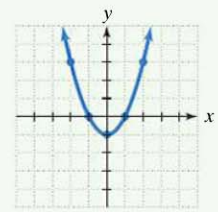
Definitions and Concepts	Examples

Section 5.1 Adding and Subtracting Polynomials (continued)

The graphs of equations defined by polynomials of degree 2, shaped like bowls or inverted bowls, can be obtained using the point-plotting method.

Graph: $y = x^2 - 1$.

x	$y = x^2 - 1$
-2	$(-2)^2 - 1 = 3$
-1	$(-1)^2 - 1 = 0$
0	$0^2 - 1 = -1$
1	$1^2 - 1 = 0$
2	$2^2 - 1 = 3$



Section 5.2 Multiplying Polynomials

Properties of Exponents

Product Rule: $b^m \cdot b^n = b^{m+n}$

Power Rule: $(b^m)^n = b^{mn}$

Products to Powers: $(ab)^n = a^n b^n$

$$x^3 \cdot x^8 = x^{3+8} = x^{11}$$
$$(x^3)^8 = x^{3 \cdot 8} = x^{24}$$
$$(-5x^2)^3 = (-5)^3(x^2)^3 = -125x^6$$

To multiply monomials, multiply coefficients and add exponents.

$$(-6x^4)(3x^{10}) = -6 \cdot 3x^{4+10} = -18x^{14}$$

To multiply a monomial and a polynomial, multiply each term of the polynomial by the monomial.

$$2x^4(3x^2 - 6x + 5)$$
$$= 2x^4 \cdot 3x^2 - 2x^4 \cdot 6x + 2x^4 \cdot 5$$
$$= 6x^6 - 12x^5 + 10x^4$$

To multiply polynomials when neither is a monomial, multiply each term of one polynomial by each term of the other polynomial. Then combine like terms.

$$(2x + 3)(5x^2 - 4x + 2)$$
$$= 2x(5x^2 - 4x + 2) + 3(5x^2 - 4x + 2)$$
$$= 10x^3 - 8x^2 + 4x + 15x^2 - 12x + 6$$
$$= 10x^3 + 7x^2 - 8x + 6$$

Section 5.3 Special Products

The FOIL method may be used when multiplying two binomials: First terms multiplied. Outside terms multiplied. Inside terms multiplied. Last terms multiplied.

F O I L

$$(3x + 7)(2x - 5) = 3x \cdot 2x + 3x(-5) + 7 \cdot 2x + 7(-5)$$
$$= 6x^2 - 15x + 14x - 35$$
$$= 6x^2 - x - 35$$

The Product of the Sum and Difference of Two Terms

$$(A + B)(A - B) = A^2 - B^2$$

$$(4x + 7)(4x - 7) = (4x)^2 - 7^2$$
$$= 16x^2 - 49$$

Definitions and Concepts	Examples

Section 5.3 Special Products (continued)

The Square of a Binomial Sum

$$(A + B)^2 = A^2 + 2AB + B^2$$

$$(x^2 + 6)^2 = (x^2)^2 + 2 \cdot x^2 \cdot 6 + 6^2 = x^4 + 12x^2 + 36$$

The Square of a Binomial Difference

$$(A - B)^2 = A^2 - 2AB + B^2$$

$$(9x - 3)^2 = (9x)^2 - 2 \cdot 9x \cdot 3 + 3^2 = 81x^2 - 54x + 9$$

Section 5.4 Polynomials in Several Variables

To evaluate a polynomial in several variables, substitute the given value for each variable and perform the resulting computation.

Evaluate $4x^2y + 3xy - 2x$ for $x = -1$ and $y = -3$.

$$\begin{aligned} &4x^2y + 3xy - 2x \\ &= 4(-1)^2(-3) + 3(-1)(-3) - 2(-1) \\ &= 4(1)(-3) + 3(-1)(-3) - 2(-1) \\ &= -12 + 9 + 2 = -1 \end{aligned}$$

For a polynomial in two variables, the degree of a term is the sum of the exponents on its variables. The degree of the polynomial is the highest degree of all its terms.

$$7x^2y + 12x^4y^3 - 17x^5 + 6$$

degree: 2 + 1 = 3 | degree: 4 + 3 = 7 | degree: 5 | degree: 0

Degree of polynomial = 7

Polynomials in several variables are added, subtracted, and multiplied using the same rules for polynomials in one variable.

$$\begin{aligned} &(5x^2y^3 - xy + 4y^2) - (8x^2y^3 - 6xy - 2y^2) \\ &= (5x^2y^3 - xy + 4y^2) + (-8x^2y^3 + 6xy + 2y^2) \\ &= (5x^2y^3 - 8x^2y^3) + (-xy + 6xy) + (4y^2 + 2y^2) \\ &= -3x^2y^3 + 5xy + 6y^2 \end{aligned}$$

F | O | I | L

$$\begin{aligned} (3x - 2y)(x - y) &= 3x \cdot x + 3x(-y) + (-2y)x + (-2y)(-y) \\ &= 3x^2 - 3xy - 2xy + 2y^2 \\ &= 3x^2 - 5xy + 2y^2 \end{aligned}$$

Section 5.5 Dividing Polynomials

Additional Properties of Exponents

Quotient Rule: $\frac{b^m}{b^n} = b^{m-n}, \quad b \neq 0$

Zero-Exponent Rule: $b^0 = 1, \quad b \neq 0$

Quotients to Powers: $\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}, \quad b \neq 0$

$$\frac{x^{12}}{x^4} = x^{12-4} = x^8$$

$$(-3)^0 = 1 \qquad -3^0 = -(3^0) = -1$$

$$\left(\frac{y^2}{4}\right)^3 = \frac{(y^2)^3}{4^3} = \frac{y^{2 \cdot 3}}{4 \cdot 4 \cdot 4} = \frac{y^6}{64}$$

To divide monomials, divide coefficients and subtract exponents.

$$\frac{-40x^{40}}{20x^{20}} = \frac{-40}{20}x^{40-20} = -2x^{20}$$

Definitions and Concepts	Examples

Section 5.5 Dividing Polynomials (continued)

To divide a polynomial by a monomial, divide each term of the polynomial by the monomial.

$$\frac{8x^6 - 4x^3 + 10x}{2x}$$
$$= \frac{8x^6}{2x} - \frac{4x^3}{2x} + \frac{10x}{2x}$$
$$= 4x^{6-1} - 2x^{3-1} + 5x^{1-1} = 4x^5 - 2x^2 + 5$$

Section 5.6 Long Division of Polynomials; Synthetic Division

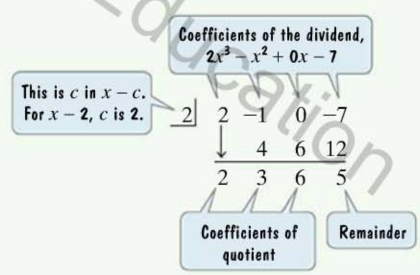
To divide a polynomial by another polynomial, begin by arranging the dividend in descending powers of the variable. If a power of a variable is missing, add that power with a coefficient of 0. Repeat the four steps—divide, multiply, subtract, bring down the next term—until the degree of the remainder is less than the degree of the divisor.

Divide: $\frac{10x^2 + 13x + 8}{2x + 3}$.

$$\begin{array}{r} 5x - 1 + \frac{11}{2x+3} \\ 2x + 3\overline{)10x^2 + 13x + 8} \\ \underline{10x^2 + 15x} \\ -2x + 8 \\ \underline{-2x - 3} \\ 11 \end{array}$$

A shortcut to long division, called synthetic division, can be used to divide a polynomial by a binomial of the form $x - c$.

Divide: $(2x^3 - x^2 - 7) \div (x - 2)$.



The answer is $2x^2 + 3x + 6 + \frac{5}{x-2}$.

Section 5.7 Negative Exponents and Scientific Notation

Negative Exponents in Numerators and Denominators

If $b \neq 0$, $b^{-n} = \frac{1}{b^n}$ and $\frac{1}{b^{-n}} = b^n$.

$$6^{-2} = \frac{1}{6^2} = \frac{1}{36}$$
$$\frac{1}{(-2)^{-4}} = (-2)^4 = 16$$
$$\left(\frac{2}{3}\right)^{-3} = \frac{2^{-3}}{3^{-3}} = \frac{3^3}{2^3} = \frac{27}{8}$$

An exponential expression is simplified when

- Each base occurs only once.
- No parentheses appear.
- No powers are raised to powers.
- No negative or zero exponents appear.

Simplify: $\frac{(2x^4)^3}{x^{18}}$.

$$\frac{(2x^4)^3}{x^{18}} = \frac{2^3(x^4)^3}{x^{18}} = \frac{8x^{4\cdot3}}{x^{18}} = \frac{8x^{12}}{x^{18}} = 8x^{12-18} = 8x^{-6} = \frac{8}{x^6}$$

Definitions and Concepts	Examples
Section 5.7 Negative Exponents and Scientific Notation (continued)	
A positive number in scientific notation is expressed as $a \times 10^n$, where $1 \le a < 10$ and n is an integer.	Write 2.9×10^{-3} in decimal notation. $2.9 \times 10^{-3} = .0029 = 0.0029$ Write 16,000 in scientific notation. $16{,}000 = 1.6 \times 10^4$
Use properties of exponents with base 10 $10^m \cdot 10^n = 10^{m+n}$, $\frac{10^m}{10^n} = 10^{m-n}$, and $(10^m)^n = 10^{mn}$ to perform computations with scientific notation.	$(5 \times 10^3)(4 \times 10^{-8})$ $= 5 \cdot 4 \times 10^{3-8}$ $= 20 \times 10^{-5}$ $= 2 \times 10^1 \times 10^{-5} = 2 \times 10^{-4}$

CHAPTER 5 REVIEW EXERCISES

5.1 *In Exercises 1–3, identify each polynomial as a monomial, binomial, or trinomial. Give the degree of the polynomial.*

1. $7x^4 + 9x$
2. $3x + 5x^2 - 2$
3. $16x$

In Exercises 4–8, add or subtract as indicated.

4. $(-6x^3 + 7x^2 - 9x + 3) + (14x^3 + 3x^2 - 11x - 7)$
5. $(9y^3 - 7y^2 + 5) + (4y^3 - y^2 + 7y - 10)$
6. $(5y^2 - y - 8) - (-6y^2 + 3y - 4)$
7. $(13x^4 - 8x^3 + 2x^2) - (5x^4 - 3x^3 + 2x^2 - 6)$
8. Subtract $x^4 + 7x^2 - 11x$ from $-13x^4 - 6x^2 + 5x$.

In Exercises 9–11, add or subtract as indicated.

9. Add.
$$\begin{array}{r} 7y^4 - 6y^3 + 4y^2 - 4y \\ \underline{y^3 - y^2 + 3y - 4} \end{array}$$
10. Subtract.
$$\begin{array}{r} 7x^2 - 9x + 2 \\ \underline{-(4x^2 - 2x - 7)} \end{array}$$
11. Subtract.
$$\begin{array}{r} 5x^3 - 6x^2 - 9x + 14 \\ -(-5x^3 + 3x^2 - 11x + 3) \end{array}$$

In Exercises 12–13, graph each equation.

12. $y = x^2 + 3$
13. $y = 1 - x^2$

5.2 *In Exercises 14–18, simplify each expression.*

14. $x^{20} \cdot x^3$
15. $y \cdot y^5 \cdot y^8$
16. $(x^{20})^5$
17. $(10y)^2$
18. $(-4x^{10})^3$

In Exercises 19–27, find each product.

19. $(5x)(10x^3)$
20. $(-12y^7)(3y^4)$
21. $(-2x^5)(-3x^4)(5x^3)$
22. $7x(3x^2 + 9)$
23. $5x^3(4x^2 - 11x)$
24. $3y^2(-7y^2 + 3y - 6)$
25. $2y^5(8y^3 - 10y^2 + 1)$
26. $(x + 3)(x^2 - 5x + 2)$
27. $(3y - 2)(4y^2 + 3y - 5)$

In Exercises 28–29, use a vertical format to find each product.

28.
$$\begin{array}{r} y^2 - 4y + 7 \\ \underline{3y - 5} \end{array}$$
29.
$$\begin{array}{r} 4x^3 - 2x^2 - 6x - 1 \\ \underline{2x + 3} \end{array}$$

5.3 *In Exercises 30–42, find each product.*

30. $(x + 6)(x + 2)$
31. $(3y - 5)(2y + 1)$
32. $(4x^2 - 2)(x^2 - 3)$