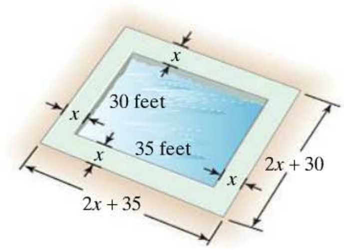
GROUP PROJECT

CHAPTER 6

Group members are on the board of a condominium association. The condominium has just installed a 35-foot-by-30-foot pool. Your job is to choose a material to surround the pool to create a border of uniform width.



a. Begin by writing an algebraic expression for the area, in square feet, of the border around the pool. (*Hint*: The border's area is the combined area of the pool and border minus the area of the pool.)

b. You must select one of the following options for the border.

Options for the Border	Price
Cement	\$6 per square foot
Outdoor carpeting	\$5 per square foot plus \$10 per foot to install edging around the rectangular border
Brick	\$8 per square foot plus a \$60 charge for delivering the bricks

Write an algebraic expression for the cost of installing the border for each of these options.

c. You would like the border to be 5 feet wide. Use the algebraic expressions in part (b) to find the cost of the border for each of the three options.

d. You would prefer not to use cement. However, the condominium association is limited by a \$5000 budget. Given this limitation, approximately how wide can the border be using outdoor carpeting or brick? Which option should you select and why?

Chapter 6 Summary

Definitions and Concepts	Examples

Section 6.1 The Greatest Common Factor and Factoring by Grouping

Definitions and Concepts	Examples
Factoring a polynomial containing the sum of monomials means finding an equivalent expression that is a product. The greatest common factor, GCF, is an expression that divides every term of the polynomial. The variable part of the GCF contains the smallest power of a variable that appears in all terms of the polynomial.	Find the GCF of $16x^2y$, $20x^3y^2$, and $8x^2y^3$. The GCF of 16, 20, and 8 is 4. The GCF of x^2, x^3, and x^2 is x^2. The GCF of y, y^2, and y^3 is y. $\text{GCF} = 4 \cdot x^2 \cdot y = 4x^2y$
To factor a monomial from a polynomial, express each term as the product of the GCF and its other factor. Then use the distributive property to factor out the GCF.	$16x^2y + 20x^3y^2 + 8x^2y^3$ $= 4x^2y \cdot 4 + 4x^2y \cdot 5xy + 4x^2y \cdot 2y^2$ $= 4x^2y(4 + 5xy + 2y^2)$
To factor a monomial from a polynomial with a negative coefficient in the first term, express each term as the product of the negative of the GCF and its other factor. Then use the distributive property to factor out the negative of the GCF.	$-20x^4y^3 + 10x^2y^2 - 15x^3y$ $= -5x^2y \cdot 4x^2y^2 - 5x^2y(-2y) - 5x^2y \cdot 3x$ The negative of the GCF is $-5x^2y$. $= -5x^2y(4x^2y^2 - 2y + 3x)$

Definitions and Concepts	Examples

Section 6.1 The Greatest Common Factor and Factoring by Grouping (continued)

To factor by grouping, factor out the GCF from each group. Then factor out the remaining common factor.	$xy + 5x - 3y - 15$ $= x(y + 5) - 3(y + 5)$ $= (y + 5)(x - 3)$

Section 6.2 Factoring Trinomials Whose Leading Coefficient Is 1

To factor a trinomial of the form $x^2 + bx + c$, find two numbers whose product is c and whose sum is b. The factorization is $(x + \text{one number})(x + \text{other number})$.	Factor: $x^2 + 9x + 20$. Find two numbers whose product is 20 and whose sum is 9. The numbers are 4 and 5. $x^2 + 9x + 20 = (x + 4)(x + 5)$

Section 6.3 Factoring Trinomials Whose Leading Coefficient Is Not 1

To factor $ax^2 + bx + c$ by trial and error, try various combinations of factors of ax^2 and c until a middle term of bx is obtained for the sum of outside and inside products.	Factor: $3x^2 + 7x - 6$. Factors of $3x^2$: $3x, x$ Factors of -6: 1 and -6, -1 and 6, 2 and -3, -2 and 3 A possible combination of these factors is $(3x - 2)(x + 3)$. Sum of outside and inside products should equal $7x$. $9x - 2x = 7x$ Thus, $3x^2 + 7x - 6 = (3x - 2)(x + 3)$.
To factor $ax^2 + bx + c$ by grouping, find the factors of ac whose sum is b. Write bx using these factors. Then factor by grouping.	Factor: $3x^2 + 7x - 6$. Find the factors of $3(-6)$, or -18, whose sum is 7. They are 9 and -2. $3x^2 + 7x - 6$ $= 3x^2 + 9x - 2x - 6$ $= 3x(x + 3) - 2(x + 3) = (x + 3)(3x - 2)$

Section 6.4 Factoring Special Forms

The Difference of Two Squares $A^2 - B^2 = (A + B)(A - B)$	$9x^2 - 25y^2$ $= (3x)^2 - (5y)^2 = (3x + 5y)(3x - 5y)$
Perfect Square Trinomials $A^2 + 2AB + B^2 = (A + B)^2$ $A^2 - 2AB + B^2 = (A - B)^2$	$x^2 + 16x + 64 = x^2 + 2 \cdot x \cdot 8 + 8^2 = (x + 8)^2$ $25x^2 - 30x + 9 = (5x)^2 - 2 \cdot 5x \cdot 3 + 3^2 = (5x - 3)^2$
Sum or Difference of Cubes $A^3 + B^3 = (A + B)(A^2 - AB + B^2)$ $A^3 - B^3 = (A - B)(A^2 + AB + B^2)$	$8x^3 - 125 = (2x)^3 - 5^3$ $= (2x - 5)[(2x)^2 + 2x \cdot 5 + 5^2]$ $= (2x - 5)(4x^2 + 10x + 25)$

Definitions and Concepts	Examples

Section 6.5 A General Factoring Strategy

A Factoring Strategy

1. Factor out the GCF.
2. a. If two terms, try

$$A^2 - B^2 = (A + B)(A - B)$$
$$A^3 + B^3 = (A + B)(A^2 - AB + B^2)$$
$$A^3 - B^3 = (A - B)(A^2 + AB + B^2).$$

   b. If three terms, try

$$A^2 + 2AB + B^2 = (A + B)^2$$
$$A^2 - 2AB + B^2 = (A - B)^2.$$

   If not a perfect square trinomial, try trial and error or grouping.

   c. If four terms, try factoring by grouping.
3. See if any factors can be factored further.
4. Check by multiplying.

Factor: $2x^4 + 10x^3 - 8x^2 - 40x$.
The GCF is $2x$.

$$2x^4 + 10x^3 - 8x^2 - 40x$$
$$= 2x(x^3 + 5x^2 - 4x - 20)$$

Four terms: Try grouping.

$$= 2x[x^2(x + 5) - 4(x + 5)]$$
$$= 2x(x + 5)(x^2 - 4)$$

This can be factored further.

$$= 2x(x + 5)(x + 2)(x - 2)$$

Section 6.6 Solving Quadratic Equations by Factoring

The Zero-Product Principle

If $AB = 0$, then $A = 0$ or $B = 0$.

Solve: $(x - 6)(x + 10) = 0$.

$$x - 6 = 0 \quad \text{or} \quad x + 10 = 0$$
$$x = 6 \qquad\qquad x = -10$$

The solutions are -10 and 6, and the solution set is $\{-10, 6\}$.

A quadratic equation in x is an equation that can be written in the standard form

$$ax^2 + bx + c = 0, \quad a \neq 0.$$

To solve by factoring, write the equation in standard form, factor, set each factor equal to zero, and solve each resulting equation. Check proposed solutions in the original equation.

Solve: $4x^2 + 9x = 9$.

$$4x^2 + 9x - 9 = 0$$
$$(4x - 3)(x + 3) = 0$$
$$4x - 3 = 0 \quad \text{or} \quad x + 3 = 0$$
$$x = \frac{3}{4} \qquad\qquad x = -3$$

The solutions are -3 and $\frac{3}{4}$, and the solution set is $\left\{-3, \frac{3}{4}\right\}$.

CHAPTER 6 REVIEW EXERCISES

6.1 *In Exercises 1–5, factor each polynomial using the greatest common factor. If there is no common factor other than 1 and the polynomial cannot be factored, so state.*

1. $30x - 45$
2. $-12x^3 - 16x^2 + 400x$
3. $30x^4y + 15x^3y + 5x^2y$
4. $7(x + 3) - 2(x + 3)$
5. $7x^2(x + y) - (x + y)$

In Exercises 6–9, factor by grouping.

6. $x^3 + 3x^2 + 2x + 6$
7. $xy + y + 4x + 4$
8. $x^3 + 5x + x^2 + 5$
9. $xy + 4x - 2y - 8$

6.2 *In Exercises 10–17, factor completely, or state that the trinomial is prime.*

10. $x^2 - 3x + 2$
11. $x^2 - x - 20$
12. $x^2 + 19x + 48$
13. $x^2 - 6xy + 8y^2$
14. $x^2 + 5x - 9$
15. $x^2 + 16xy - 17y^2$