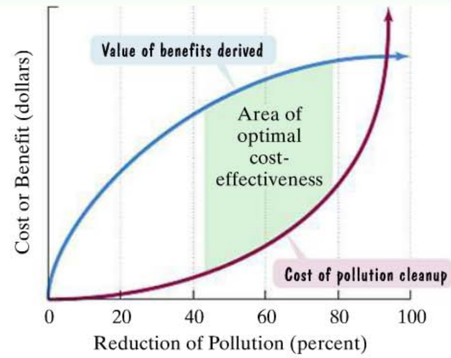
GROUP PROJECT

CHAPTER 7

A cost-benefit analysis compares the estimated costs of a project with the benefits that will be achieved. Costs and benefits are given monetary values and compared using a benefit-cost ratio. As shown in the figure, a favorable ratio for a project means that the benefits outweigh the costs and the project is cost-effective. As a group, select an environmental project that was implemented in your area of the country. Research the cost and benefit graphs that resulted in the implementation of this project. How were the benefits converted into monetary terms? Is there an equation for either the cost model or the benefit model? Group members may need to interview members of environmental groups and businesses that were part of this project. You may wish to consult an environmental science textbook to find out more about cost-benefit analyses. After doing your research, the group should write or present a report explaining why the cost-benefit analysis resulted in the project's implementation.



Chapter 7 Summary

Definitions and Concepts	Examples

Section 7.1 Rational Expressions and Their Simplification

A rational expression is the quotient of two polynomials. To find values for which a rational expression is undefined, set the denominator equal to 0 and solve.

Find all numbers for which

$$\frac{7x}{x^2 - 3x - 4}$$

is undefined.

$$x^2 - 3x - 4 = 0$$
$$(x - 4)(x + 1) = 0$$
$$x - 4 = 0 \quad \text{or} \quad x + 1 = 0$$
$$x = 4 \qquad\qquad x = -1$$

The expression is undefined for 4 and -1.

To simplify a rational expression:

1. Factor the numerator and the denominator completely.
2. Divide the numerator and the denominator by any common factors.

If factors in the numerator and denominator are opposites, their quotient is -1.

Simplify: $\frac{3x + 18}{x^2 - 36}$.

$$\frac{3x + 18}{x^2 - 36} = \frac{3\overset{1}{\cancel{(x + 6)}}}{\underset{1}{\cancel{(x + 6)}}(x - 6)} = \frac{3}{x - 6}$$

Definitions and Concepts	Examples

Section 7.2 Multiplying and Dividing Rational Expressions

Multiplying Rational Expressions

1. Factor completely.
2. Divide numerators and denominators by common factors.
3. Multiply remaining factors in the numerators and multiply the remaining factors in the denominators.

$$\frac{x^2+3x-10}{x^2-2x}\cdot\frac{x^2}{x^2-25}$$
$$=\frac{\overset{1}{\cancel{(x+5)}}\overset{1}{\cancel{(x-2)}}}{\underset{1}{\cancel{x}}\underset{1}{\cancel{(x-2)}}}\cdot\frac{\overset{1}{\cancel{x}}\cdot x}{\underset{1}{\cancel{(x+5)}}(x-5)}$$
$$=\frac{x}{x-5}$$

Dividing Rational Expressions

Invert the divisor and multiply.

$$\frac{3y+3}{(y+2)^2}\div\frac{y^2-1}{y+2}$$
$$=\frac{3y+3}{(y+2)^2}\cdot\frac{y+2}{y^2-1}$$
$$=\frac{3\overset{1}{\cancel{(y+1)}}}{(y+2)\underset{1}{\cancel{(y+2)}}}\cdot\frac{\overset{1}{\cancel{(y+2)}}}{\underset{1}{\cancel{(y+1)}}(y-1)}$$
$$=\frac{3}{(y+2)(y-1)}$$

Section 7.3 Adding and Subtracting Rational Expressions with the Same Denominator

To add or subtract rational expressions with the same denominator, add or subtract the numerators and place the result over the common denominator. If possible, factor and simplify the resulting expression.

$$\frac{y^2-3y+4}{y^2+8y+15}-\frac{y^2-5y-2}{y^2+8y+15}$$
$$=\frac{y^2-3y+4-(y^2-5y-2)}{y^2+8y+15}$$
$$=\frac{y^2-3y+4-y^2+5y+2}{y^2+8y+15}$$
$$=\frac{2y+6}{(y+5)(y+3)}$$
$$=\frac{2\overset{1}{\cancel{(y+3)}}}{(y+5)\underset{1}{\cancel{(y+3)}}}=\frac{2}{y+5}$$

To add or subtract rational expressions with opposite denominators, multiply either rational expression by $\frac{-1}{-1}$ to obtain a common denominator.

$$\frac{7}{x-6}+\frac{x+4}{6-x}$$
$$=\frac{7}{x-6}+\frac{(-1)}{(-1)}\cdot\frac{x+4}{6-x}$$
$$=\frac{7}{x+6}+\frac{-x-4}{x-6}$$
$$=\frac{7-x-4}{x-6}=\frac{3-x}{x-6}$$

Definitions and Concepts	Examples

Section 7.4 Adding and Subtracting Rational Expressions with Different Denominators

Finding the Least Common Denominator (LCD)

1. Factor denominators completely.
2. List factors of the first denominator.
3. Add to the list any factors of the second denominator that are not already in the list.
4. The LCD is the product of factors in step 3.

Find the LCD of

$$\frac{x+1}{2x-2} \quad \text{and} \quad \frac{2x}{x^2+2x-3}.$$

$$2x - 2 = 2(x-1)$$

$$x^2 + 2x - 3 = (x-1)(x+3)$$

Factors of first denominator: $2, x - 1$

Factors of second denominator not in the list: $x + 3$

LCD: $2(x-1)(x+3)$

Adding and Subtracting Rational Expressions with Different Denominators

1. Find the LCD.
2. Rewrite each rational expression as an equivalent expression with the LCD.
3. Add or subtract numerators, placing the resulting expression over the LCD.
4. If possible, simplify.

$$\frac{x+1}{2x-2} - \frac{2x}{x^2+2x-3}$$

$$= \frac{x+1}{2(x-1)} - \frac{2x}{(x-1)(x+3)}$$

LCD is $2(x-1)(x+3)$.

$$= \frac{(x+1)(x+3)}{2(x-1)(x+3)} - \frac{2x \cdot 2}{2(x-1)(x+3)}$$

$$= \frac{x^2+4x+3-4x}{2(x-1)(x+3)}$$

$$= \frac{x^2+3}{2(x-1)(x+3)}$$

Section 7.5 Complex Rational Expressions

Complex rational expressions have numerators or denominators containing one or more rational expressions. Complex rational expressions can be simplified by obtaining single expressions in the numerator and denominator and then dividing. They can also be simplified by multiplying the numerator and denominator by the LCD of all rational expressions within the complex rational expression.

Simplify by dividing: $\dfrac{\frac{1}{x}+5}{\frac{1}{x}-\frac{1}{3}}$.

$$= \frac{\frac{1}{x}+\frac{5x}{x}}{\frac{3}{3x}-\frac{x}{3x}} = \frac{\frac{1+5x}{x}}{\frac{3-x}{3x}} = \frac{1+5x}{\cancel{x}_1} \cdot \frac{3\cancel{x}^1}{3-x}$$

$$= \frac{3(1+5x)}{3-x} \quad \text{or} \quad \frac{3+15x}{3-x}$$

Simplify by the LCD method: $\dfrac{\frac{1}{x}+5}{\frac{1}{x}-\frac{1}{3}}$.

LCD is $3x$.

$$\frac{3x}{3x} \cdot \frac{\left(\frac{1}{x}+5\right)}{\left(\frac{1}{x}-\frac{1}{3}\right)} = \frac{3x \cdot \frac{1}{x} + 3x \cdot 5}{3x \cdot \frac{1}{x} - 3x \cdot \frac{1}{3}}$$

$$= \frac{3+15x}{3-x}$$

Definitions and Concepts	Examples

Section 7.6 Solving Rational Equations

A rational equation is an equation containing one or more rational expressions.

Solving Rational Equations

1. List restrictions on the variable.
2. Clear fractions by multiplying both sides by the LCD.
3. Solve the resulting equation.
4. Reject any proposed solution in the list of restrictions. Check other proposed solutions in the original equation.

Solve: $\frac{7x}{x^2-4}+\frac{5}{x-2}=\frac{2x}{x^2-4}$.

$$\frac{7x}{(x+2)(x-2)}+\frac{5}{x-2}=\frac{2x}{(x+2)(x-2)}$$

Denominators would equal 0 if $x=-2$ or $x=2$.
Restrictions: $x \neq -2$ and $x \neq 2$.

LCD is $(x+2)(x-2)$.

$$(x+2)(x-2)\left[\frac{7x}{(x+2)(x-2)}+\frac{5}{x-2}\right]=(x+2)(x-2)\cdot\frac{2x}{(x+2)(x-2)}$$

$$7x+5(x+2)=2x$$
$$7x+5x+10=2x$$
$$12x+10=2x$$
$$10=-10x$$
$$-1=x$$

The proposed solution, -1, is not part of the restriction $x \neq -2$ and $x \neq 2$. It checks. The solution is -1 and the solution set is $\{-1\}$.

To solve a formula for a variable, get the specified variable alone on one side of the formula. When working with formulas containing rational expressions, it is sometimes necessary to factor out the variable you are solving for.

Solve: $\frac{e}{E}=\frac{r}{r+R}$ for r.

$$E(r+R)\cdot\frac{e}{E}=E(r+R)\cdot\frac{r}{r+R}$$ LCD is $E(r+R)$.

$$e(r+R)=Er$$
$$er+eR=Er$$
$$eR=Er-er$$
$$eR=(E-e)r$$
$$\frac{eR}{E-e}=r$$

Definitions and Concepts	Examples

Section 7.7 Applications Using Rational Equations and Proportions

Motion problems involving time are solved using

$$t = \frac{d}{r}.$$

$$\text{Time traveled} = \frac{\text{Distance traveled}}{\text{Rate of travel}}$$

It takes a cyclist who averages 16 miles per hour in still air the same time to travel 48 miles with the wind as 16 miles against the wind. What is the wind's rate?

$$x = \text{wind's rate}$$

$$16 + x = \text{cyclist's rate with wind}$$

$$16 - x = \text{cyclist's rate against wind}$$

	Distance	Rate	Time = $\frac{\text{Distance}}{\text{Rate}}$
With wind	48	$16 + x$	$\frac{48}{16 + x}$
Against wind	16	$16 - x$	$\frac{16}{16 - x}$

Two times are equal.

$$\frac{48}{16 + x} = \frac{16}{16 - x}$$

$$(16 + x)(16 - x) \cdot \frac{48}{16 + x} = \frac{16}{16 - x} \cdot (16 + x)(16 - x)$$

$$48(16 - x) = 16(16 + x)$$

Solving this equation, $x = 8$.

The wind's rate is 8 miles per hour.

Work problems are solved using the following condition:

Fraction of job done by the first + fraction of job done by the second = 1.

One pipe fills a pool in 20 hours and a second pipe in 15 hours. How long will it take to fill the pool using both pipes?

$$x = \text{time using both pipes}$$

Fraction of pool filled by pipe 1 in x hours + fraction of pool filled by pipe 2 in x hours = 1.

$$\frac{x}{20} + \frac{x}{15} = 1$$

$$60\left(\frac{x}{20} + \frac{x}{15}\right) = 60 \cdot 1$$

$$3x + 4x = 60$$

$$7x = 60$$

$$x = \frac{60}{7} = 8\frac{4}{7} \text{ hours}$$

It will take $8\frac{4}{7}$ hours for both pipes to fill the pool.

Definitions and Concepts	Examples

Section 7.7 Applications Using Rational Equations and Proportions (continued)

A proportion is a statement in the form $\frac{a}{b} = \frac{c}{d}$. The cross-products principle states that if $\frac{a}{b} = \frac{c}{d}$, then $ad = bc$ ($b \neq 0$ and $d \neq 0$).

Solving Applied Problems Using Proportions

1. Read the problem and represent the unknown quantity by x (or any letter).
2. Set up a proportion by listing the given ratio on one side and the ratio with the unknown quantity on the other side.
3. Drop units and apply the cross-products principle.
4. Solve for x and anwer the question.

30 elk are tagged and released. Sometime later, a sample of 80 elk is observed and 10 are tagged. How many elk are there?

$$x = \text{number of elk}$$

Tagged / Total:

$$\frac{30}{x} = \frac{10}{80}$$

$$10x = 30 \cdot 80$$

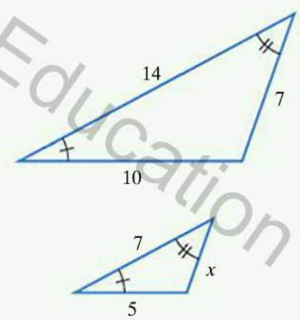
$$10x = 2400$$

$$x = 240$$

There are 240 elk.

Similar triangles have the same shape, but not necessarily the same size. Corresponding angles have the same measure, and corresponding sides are proportional. If the measures of two angles of one triangle are equal to those of two angles of a second triangle, then the two triangles are similar.

Find x for these similar triangles.



Corresponding sides are proportional:

$$\frac{7}{x} = \frac{10}{5}. \quad \left(\text{or } \frac{7}{x} = \frac{14}{7}\right)$$

$$5x \cdot \frac{7}{x} = \frac{10}{5} \cdot 5x$$

$$35 = 10x$$

$$x = \frac{35}{10} = 3.5$$