Chapter 8 Summary

Definitions and Concepts	Examples

Section 8.1 Introduction to Functions

A relation is any set of ordered pairs. The set of first components of the ordered pairs is the domain and the set of second components is the range. A function is a relation in which each member of the domain corresponds to exactly one member of the range. No two ordered pairs of a function can have the same first component and different second components.

The domain of the relation $\{(1, 2), (3, 4), (3, 7)\}$ is $\{1, 3\}$. The range is $\{2, 4, 7\}$. The relation is not a function: 3, in the domain, corresponds to both 4 and 7 in the range.

If a function is defined by an equation, the notation $f(x)$, read "f of x" or "f at x," describes the value of the function at the number, or input, x.

If $f(x) = 7x - 5$, then

$$\begin{aligned} f(a+2) &= 7(a+2) - 5 \\ &= 7a + 14 - 5 \\ &= 7a + 9. \end{aligned}$$

Section 8.2 Graphs of Functions

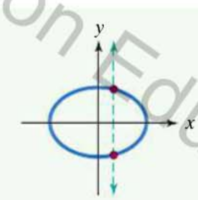
The graph of a function is the graph of its ordered pairs.

The Vertical Line Test for Functions

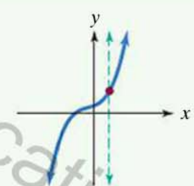
If any vertical line intersects a graph in more than one point, the graph does not define y as a function of x.

At the left or right of a function's graph, you will often find closed dots, open dots, or arrows. A closed dot shows that the graph ends and the point belongs to the graph. An open dot shows that the graph ends and the point does not belong to the graph. An arrow indicates that the graph extends indefinitely.

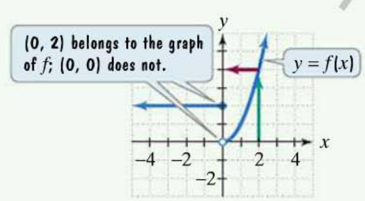
The graph of a function can be used to determine the function's domain and its range. To find the domain, look for all the inputs on the x-axis that correspond to points on the graph. To find the range, look for all the outputs on the y-axis that correspond to points on the graph.



Not the graph of a function



The graph of a function



To find $f(2)$, locate 2 on the x-axis. The graph shows $f(2) = 4$.

Domain of $f = (-\infty, \infty)$

Range of $f = (0, \infty)$

Section 8.3 The Algebra of Functions

A Function's Domain

If a function f does not model data or verbal conditions, its domain is the largest set of real numbers for which the value of $f(x)$ is a real number. Exclude from a function's domain real numbers that cause division by zero and real numbers that result in a square root of a negative number.

$$f(x) = 7x + 13$$

Domain of $f = (-\infty, \infty)$

$$g(x) = \frac{7x}{12 - x}$$

Domain of $g = (-\infty, 12)$ or $(12, \infty)$

Definitions and Concepts	Examples

Section 8.3 The Algebra of Functions (continued)

The Algebra of Functions

Let f and g be two functions. The sum $f + g$, the difference $f - g$, the product fg, and the quotient $\frac{f}{g}$ are functions whose domains are the set of all real numbers common to the domains of f and g, defined as follows:

1. Sum: $(f + g)(x) = f(x) + g(x)$
2. Difference: $(f - g)(x) = f(x) - g(x)$
3. Product: $(fg)(x) = f(x) \cdot g(x)$
4. Quotient: $\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)}, g(x) \neq 0.$

Let $f(x) = x^2 + 2x$ and $g(x) = 4 - x$.

- $(f + g)(x) = (x^2 + 2x) + (4 - x) = x^2 + x + 4$
  $(f + g)(-2) = (-2)^2 + (-2) + 4 = 4 - 2 + 4 = 6$
- $(f - g)(x) = (x^2 + 2x) - (4 - x) = x^2 + 2x - 4 + x$
  $= x^2 + 3x - 4$
  $(f - g)(5) = 5^2 + 3 \cdot 5 - 4 = 25 + 15 - 4 = 36$
- $(fg)(x) = (x^2 + 2x)(4 - x) = 4x^2 - x^3 + 8x - 2x^2$
  $= -x^3 + 2x^2 + 8x$
  $(fg)(1) = -1^3 + 2 \cdot 1^2 + 8 \cdot 1 = -1 + 2 + 8 = 9$
- $\left(\frac{f}{g}\right)(x) = \frac{x^2 + 2x}{4 - x}, x \neq 4$
  $\left(\frac{f}{g}\right)(3) = \frac{3^2 + 2 \cdot 3}{4 - 3} = \frac{9 + 6}{1} = 15$

Section 8.4 Composite and Inverse Functions

Composite Functions

The composite function $f \circ g$ is defined by

$$(f \circ g)(x) = f(g(x)).$$

The composite function $g \circ f$ is defined by

$$(g \circ f)(x) = g(f(x)).$$

Let $f(x) = x^2 + x$ and $g(x) = 2x + 1$.

- $(f \circ g)(x) = f(g(x)) = (g(x))^2 + g(x)$
  Replace x with $g(x)$.
  $= (2x + 1)^2 + (2x + 1) = 4x^2 + 4x + 1 + 2x + 1$
  $= 4x^2 + 6x + 2$
- $(g \circ f)(x) = g(f(x)) = 2f(x) + 1$
  Replace x with $f(x)$.
  $= 2(x^2 + x) + 1 = 2x^2 + 2x + 1$

Inverse Functions

If $f(g(x)) = x$ and $g(f(x)) = x$, function g is the inverse of function f, denoted f^{-1} and read "f inverse." The procedure for finding a function's inverse uses a switch-and-solve strategy. Switch x and y, then solve for y.

If $f(x) = 2x - 5$, find $f^{-1}(x)$.

$y = 2x - 5$ Replace $f(x)$ with y.

$x = 2y - 5$ Exchange x and y.

$x + 5 = 2y$ Solve for y.

$\frac{x + 5}{2} = y$

$f^{-1}(x) = \frac{x + 5}{2}$ Replace y with $f^{-1}(x)$.

The Horizontal Line Test for Inverse Functions

A function, f, has an inverse that is a function, f^{-1}, if there is no horizontal line that intersects the graph of f at more than one point. A one-to-one function is one in which no two different ordered pairs have the same second component. Only one-to-one functions have inverse functions. If the point (a, b) is on the graph of f, then the point (b, a) is on the graph of f^{-1}. The graph of f^{-1} is a reflection of the graph of f about the line $y = x$.