97. Solve by graphing:

$$\begin{cases} y = 3x - 2 \\ y = -2x + 8. \end{cases}$$

(Section 4.1, Example 3)

98. Factor completely: $2x^6 + 20x^5y + 50x^4y^2$.
(Section 6.5, Example 8)

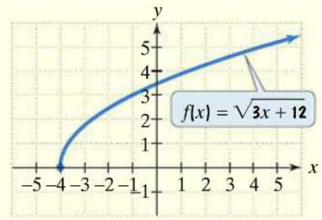
Preview Exercises

Exercises 99–101 will help you prepare for the material covered in the first section of the next chapter.

99. If $f(x) = \sqrt{3x + 12}$, find $f(-1)$.

100. If $f(x) = \sqrt{3x + 12}$, find $f(8)$.

101. Use the graph of $f(x) = \sqrt{3x + 12}$, to identify the function's domain and its range.



GROUP PROJECT

CHAPTER 9

Each group member should research one situation that provides two different pricing options. These can involve areas such as public transportation options (with or without discount passes) or telephone plans or anything of interest. Be sure to bring in all the details for each option. At the group meeting, select the two pricing situations that are most interesting and relevant. Using each situation, write a word problem about selecting the better of the two options. The word problem should be one that can be solved using a linear inequality. The group should turn in the two problems and their solutions.

Chapter 9 Summary

Definitions and Concepts	Examples

Section 9.1 Reviewing Linear Inequalities and Using Inequalities in Business Applications

A linear inequality in one variable can be written in the form $ax + b < 0$, $ax + b \le 0$, $ax + b > 0$, or $ax + b \ge 0$. The set of all numbers that make the inequality a true statement is its solution set, represented using interval notation.

Solving a Linear Inequality

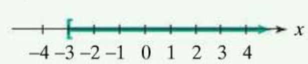
1. Simplify each side.
2. Collect variable terms on one side and constant terms on the other side.
3. Isolate the variable and solve.

If an inequality is multiplied or divided by a negative number, the direction of the inequality symbol must be reversed.

Solve: $2(x + 3) - 5x \le 15$.

$$2x + 6 - 5x \le 15$$
$$-3x + 6 \le 15$$
$$-3x \le 9$$
$$\frac{-3x}{-3} \ge \frac{9}{-3}$$
$$x \ge -3$$

Solution set: $[-3, \infty)$



Definitions and Concepts	Examples

Section 9.1 Reviewing Linear Inequalities and Using Inequalities in Business Applications (continued)

Functions of Business

A company produces and sells x units of a product.

Revenue Function

$$R(x) = (\text{price per unit sold})x$$

Cost Function

$$C(x) = \text{fixed cost} + (\text{cost per unit produced})x$$

Profit Function

$$P(x) = R(x) - C(x)$$

A business makes money, or has a gain, when $P(x) > 0$.
A business loses money, or has a loss, when $P(x) < 0$.

A company that manufactures lamps has a fixed cost of $80,000 and it costs $20 to produce each lamp. Lamps are sold for $70.

a. Write the cost function.

$$C(x) = 80{,}000 + 20x$$

Fixed cost

Variable cost: $20 per lamp.

b. Write the revenue function.

$$R(x) = 70x$$

Revenue per lamp, $70, times number of lamps sold

c. Write the profit function.

$$P(x) = R(x) - C(x)$$
$$= 70x - (80{,}000 + 20x)$$
$$= 50x - 80{,}000$$

d. More than how many lamps must be produced and sold for the company to make money?

Solve $P(x) > 0$.

$$50x - 80{,}000 > 0$$
$$50x > 80{,}000$$
$$x > 1600$$

More than 1600 lamps must be produced and sold for the company to make money.

Section 9.2 Compound Inequalities

Intersection (∩) and Union (∪)

$A \cap B$ is the set of elements common to both set A and set B.
$A \cup B$ is the set of elements that are members of set A or set B or of both sets.

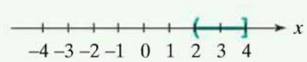
$\{1, 3, 5, 7\} \cap \{5, 7, 9, 11\} = \{5, 7\}$
$\{1, 3, 5, 7\} \cup \{5, 7, 9, 11\} = \{1, 3, 5, 7, 9, 11\}$

A compound inequality is formed by joining two inequalities with the word *and* or *or*.

When the connecting word is *and*, graph each inequality separately and take the intersection of their solution sets.

Solve: $x + 1 > 3$ and $x + 4 \le 8$.

$x > 2$ and $x \le 4$



Solution set: $(2, 4]$

The compound inequality $a < x < b$ means $a < x$ and $x < b$. Solve by isolating the variable in the middle.

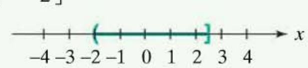
Solve: $-1 < \dfrac{2x + 1}{3} \le 2$.

$-3 < 2x + 1 \le 6$ Multiply by 3.

$-4 < 2x \le 5$ Subtract 1.

$-2 < x \le \dfrac{5}{2}$ Divide by 2.

Solution set: $\left(-2, \dfrac{5}{2}\right]$



Definitions and Concepts | **Examples**

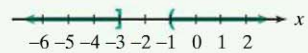
Section 9.2 Compound Inequalities (continued)

When the connecting word in a compound inequality is *or*, graph each inequality separately and take the union of their solution sets.

Solve: $x - 2 > -3$ or $2x \le -6$.

$x > -1$ or $x \le -3$

Solution set: $(-\infty, -3] \cup (-1, \infty)$



Section 9.3 Equations and Inequalities Involving Absolute Value

Absolute Value Equations

1. If $c > 0$, then $|u| = c$ means $u = c$ or $u = -c$.
2. If $c < 0$, then $|u| = c$ has no solution.
3. If $c = 0$, then $|u| = 0$ means $u = 0$.

Solve: $|2x - 7| = 3$.

$$2x - 7 = 3 \quad \text{or} \quad 2x - 7 = -3$$
$$2x = 10 \qquad 2x = 4$$
$$x = 5 \qquad x = 2$$

The solution set is $\{2, 5\}$.

Absolute Value Equations with Two Absolute Value Bars

If $|u| = |v|$, then $u = v$ or $u = -v$.

Solve: $|x - 6| = |2x + 1|$.

$$x - 6 = 2x + 1 \quad \text{or} \quad x - 6 = -(2x + 1)$$
$$-x - 6 = 1 \qquad x - 6 = -2x - 1$$
$$-x = 7 \qquad 3x - 6 = -1$$
$$x = -7 \qquad 3x = 5$$
$$x = \frac{5}{3}$$

The solutions are -7 and $\frac{5}{3}$, and the solution set is $\left\{-7, \frac{5}{3}\right\}$.

Solving Absolute Value Inequalities

If c is a positive number,

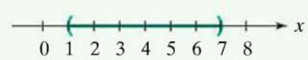
1. $|u| < c$ is equivalent to $-c < u < c$.
2. $|u| > c$ is equivalent to $u < -c$ or $u > c$.

In each case, the absolute value inequality is rewritten as an equivalent compound inequality without absolute value bars.

Solve: $|x - 4| < 3$.

$$-3 < x - 4 < 3$$
$$1 < x < 7 \quad \text{Add 4.}$$

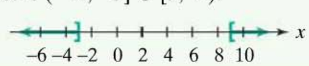
The solution set is $(1, 7)$.



Solve: $\left|\frac{x}{3} - 1\right| \ge 2$.

$$\frac{x}{3} - 1 \le -2 \quad \text{or} \quad \frac{x}{3} - 1 \ge 2.$$
$$x - 3 \le -6 \quad \text{or} \quad x - 3 \ge 6 \quad \text{Multiply by 3.}$$
$$x \le -3 \quad \text{or} \quad x \ge 9 \quad \text{Add 3.}$$

The solution set is $(-\infty, -3] \cup [9, \infty)$.



Absolute Value Inequalities with Unusual Solution Sets

If c is a negative number,

1. $|u| < c$ has no solution.
2. $|u| > c$ is true for all real numbers for which u is defined.

- $|x - 4| < -3$ has no solution. The solution set is $\varnothing$.
- $|3x + 6| > -12$ is true for all real numbers. The solution set is $(-\infty, \infty)$.

Definitions and Concepts	Examples

Section 9.4 Linear Inequalities in Two Variables

If the equal sign in $Ax + By = C$ is replaced with an inequality symbol, the result is a linear inequality in two variables. Its graph is the set of all points whose coordinates satisfy the inequality. To obtain the graph,

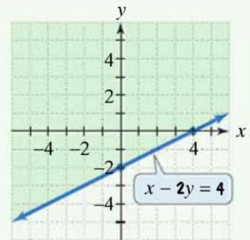
1. Replace the inequality symbol with an equal sign and graph the boundary line. Use a solid line for $\le$ or $\ge$ and a dashed line for $<$ or $>$.
2. Choose a test point not on the line and substitute its coordinates into the inequality.
3. If a true statement results, shade the half-plane containing the test point. If a false statement results, shade the half-plane not containing the test point.

Graph: $x - 2y \le 4$.

1. Graph $x - 2y = 4$. Use a solid line because the inequality symbol is $\le$.
2. Test $(0, 0)$.

$$x - 2y \le 4$$
$$0 - 2 \cdot 0 \overset{?}{\le} 4$$
$$0 \le 4, \quad \text{true}$$

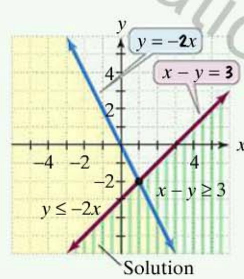
3. The inequality is true. Shade the half-plane containing $(0, 0)$.



Two or more linear inequalities make up a system of linear inequalities. A solution is an ordered pair satisfying all inequalities in the system. To graph a system of inequalities, graph each inequality in the system. The overlapping region, if there is one, represents the solutions of the system. If there is no overlapping region, the system has no solution.

Graph the solutions of the system:

$$\begin{cases} y \le -2x \\ x - y \ge 3. \end{cases}$$



CHAPTER 9 REVIEW EXERCISES

9.1 *In Exercises 1–5, solve each linear inequality and graph the solution set on a number line.*

1. $-6x + 3 \le 15$
2. $6x - 9 \ge -4x - 3$
3. $\frac{x}{3} - \frac{3}{4} - 1 > \frac{x}{2}$
4. $6x + 5 > -2(x - 3) - 25$
5. $3(2x - 1) - 2(x - 4) \ge 7 + 2(3 + 4x)$
6. The cost and revenue functions for producing and selling x units of a toaster oven are

$$C(x) = 40x + 357{,}000 \quad \text{and} \quad R(x) = 125x.$$

a. Write the profit function, P, from producing and selling x toaster ovens.

b. More than how many toaster ovens must be produced and sold for the business to make money?

Use this information to solve Exercises 7–10: A company is planning to produce and sell a new line of computers. The fixed cost will be \$360,000 and it will cost \$850 to produce each computer. Each computer will be sold for \$1150.

7. Write the cost function, C, of producing x computers.
8. Write the revenue function, R, from the sale of x computers.