Mini Lecture 9.1
Reviewing Linear Inequalities and Using Inequalities in Business Applications

Learning Objectives:

1. Review how to solve linear inequalities.
2. Use linear inequalities to solve problems involving revenue, cost, and profit.

Examples:

1. a. $2x - 5 \geq 3$ b. $3x - 5 \leq 6x + 4$ c. $\frac{x+1}{4} > \frac{2x-1}{4} + \frac{3}{8}$

   d. $4(x + 1) > 4x + 2$ e. $2x + 2 \leq 2x - 2$

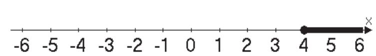
2. Use the revenue and cost functions
   $R(x) = 100x$
   $C(x) = 160{,}000 + 75x$
   to write the profit function for producing and selling x units.

With the profit function, determine how many units must be produced and sold for the business to make money.

Teaching Notes:

- When multiplying or dividing both sides of an inequality by a negative quantity, remember to reverse the direction of the inequality symbol.
- When an inequality has been solved and the variable has been eliminated and the result is a false statement, the inequality has no solution, Ø.
- When an inequality has been solved and the variable has been eliminated and the result is a true statement, the solution for the inequality is all real numbers.
- Revenue and cost function: A company produces and sells x units of a product. The Revenue Function is: $R(x)$ = (price per unit sold)x and the Cost Function is: $C(x)$ = fixed cost + (cost per unit produced)x.
- The Profit, $P(x)$, generated after producing and selling x units of a product is given by the profit function $P(x) = R(x) = C(x)$ where R and C are the revenue and cost functions, respectively.

Answers: 1.a. $\{x \mid x \geq 4\}$ or $[4, \infty)$

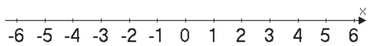


b. $\{x \mid x \geq -3\}$ or $[-3, \infty)$

c. $\{x \mid x < \frac{1}{2}\}$ or $(-\infty, \frac{1}{2})$

d. $\{x \mid x \text{ is a real number}\}$ or $(-\infty, \infty)$

e. Ø



2. $P(x) = 25x - 160{,}000$; more than 6400 units

Mini Lecture 9.2
Compound Inequalities

Learning Objectives:

1. Find the intersection of two sets.
2. Solve compound inequalities involving *and*.
3. Find the union of two sets.
4. Solve compound inequalities involving *or*.

Examples:

1. Find the intersection of the sets.
   a. $\{-3, -1, 3, 5\} \cap \{-5, -3, 1, 3\}$ b. $\{0, 1, 2, 3, 4\} \cap \{2, 3, 5, 6\}$
   c. $\{2, 4, 6, 8\} \cap \{1, 3, 5, 7\}$

2. Solve. Give solutions in interval notation.
   a. $x + 1 < 6$ and $x + 5 > 8$ b. $-4x - 5 < 3$ and $x + 1 < 5$
   c. $-3 \le 2x - 5 < 3$ d. $3 < \frac{x}{2} + 5 \le 6$

3. Find the union of the sets.
   a. $\{-8, -7, -6\} \cup \{-5, -4, -3\}$ b. $\{2, 4, 6, 8\} \cup \{1, 3, 5, 7\}$
   c. {dogs) $\cup$ {cats}

4. Solve. Give solutions in interval notation.
   a. $x + 5 \le -2$ or $x + 5 \ge 2$ b. $4x + 2 < -10$ or $5 - 2x < 9$
   c. $2x + 5 > 3x - 1$ or $x - 4 < 2x + 6$

5. Solve. Graph. Give solution in interval notation.
   a. $-6 < 2x + 2 \le 14$ b. $5x - 3 \le x + 1$ or $-8x \le -16$
   c. $3(x + 1) < 2(x + 2)$ or $2(x - 1) \ge x + 2$ d. $2x + 4 < -8$ and $3x + 5 > 8$

Teaching Notes:

- It can be helpful when graphing compound inequalities involving *and* to graph each inequality on the same number line with different colored highlighters. This will enable the students to "see" the intersection of the two graphs.
- Students will again need to be reminded that when multiplying both sides of an equality by a negative number, the inequality sign must be reversed.
- Compound inequalities involving *or* indicate intersection.
- Compound inequalities involving *and* indicate union.

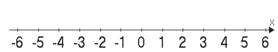
Answers: 1. a. $\{-3, 3\}$ b. $\{2, 3\}$ c. Ø 2. a. x; $(3, 5)$ b. $(-2, 4)$ c. $[1, 4)$ d. $(14, 12]$
3. a. $\{-8, -7, -6, -5, -4, -3\}$ b. $\{1, 2, 3, 4, 5, 6, 7, 8\}$ c. {dogs, cats} 4. a. $(-\infty, -7] \cup [-3, \infty)$
b. x; $(-\infty, -3) \cup (-2, \infty)$

c. $(-\infty, \infty)$ 5. a.

; $(-4, 6]$ b. 

; $(-\infty, 1] \cup [2, \infty)$

c. 

; $(-\infty, 1) \cup [4, \infty)$ d. 

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Mini Lecture 9.3

Equations and Inequalities Involving Absolute Value

Learning Objectives:

1. Solve absolute value equations.
2. Solve absolute value inequalities of the form $|u| < c$.
3. Solve absolute value inequalities of the form $|u| > c$.
4. Recognize absolute value inequalities with no solution or all real numbers as solutions.
5. Solve problems using absolute value inequalities.

Examples:

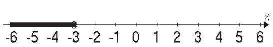
1. Solve.
   a. $|3x-2| = 5$ b. $2|y+4| = 8$
   c. $|x-2|+4 = 2$ d. $|2x+1| = |3x+4|$

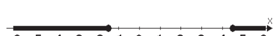
2. Solve and graph the solution set on a number line. Give solutions in interval notation.
   a. $|x+4.5| < 1.5$ b. $|2x-3| \geq 6$
   c. $|3x+4| < -2$ d.. $|2x-2| > -4$

Teaching Notes:

- The absolute value of a, denoted $|a|$ is the distance from 0 to a on a number line.
- If c is a positive real number and x is an algebraic expression then rewrite an absolute value equation without absolute value bars. $|x| = c$ is equivalent to $x = \pm c$
- If c is a positive number and x is an algebraic expression then $|x| > c$ are the numbers $-c < x < c$. $|x| > c$ are the numbers $x < -c$ or $x > c$. These rules are true for $\leq$ or $\geq$ respectively.
- If c is a negative number and x is an algebraic expression then $|x| < c$ has no solution and $|x| > c$ is true for all real numbers for which x is defined.

Answers: 1. a. -1 and $\frac{7}{3}$ or $\{-1, \frac{7}{3}\}$ b. 0 and -8 or $\{0, -8\}$ c. Ø d. -3 and -1 or $\{-3, -1\}$

2. a. $(-6, -3)$ 

b. $(-\infty, \frac{-3}{2}] \cup [\frac{9}{2}, \infty)$ 

c. Ø

d. $\{x \mid x$ is a real number$\}$ or $(-\infty, \infty)$