

Linear Inequalities in One Variable

Professor Tim Busken

Department of Mathematics

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Linear Inequalities in One Variable

Learning Objectives:

- Solve a linear inequality in one variable and graph the solution set.
- Write solutions to inequalities using interval notation.
- Solve a compound inequality and graph the solution set.
- Solve application problems using inequalities.

Linear Inequalities in One Variable

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 - 2 less than or equal to \leq ,
 - 3 greater than $>$,
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Linear Inequalities in One Variable

Definition

Replacing the equal sign in the general linear equation $a \cdot x + b = c$ by any of the symbols $<$, \leq , $>$ or \geq gives a **linear inequality in one variable**.

For example, $2 \cdot x - 1 \leq 0$ and $3x + 5 > 8$ are two different linear inequalities in a single variable, x .

Solving Linear Inequalities

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For example, $\{x \mid x < -2\}$ is shorthand notation for the set of real numbers less than -2 .

$\{x \mid x < -2\}$
↙ ↓ ↘
x such that x is any real number less than -2

Addition Property for Inequalities

For any three algebraic expressions A , B and C ,

$$\text{If } A < B$$

$$\text{then } A + C < B + C$$

In words: Adding the same quantity to both sides of an inequality will not change the solution set.

We can use the Addn. Prop. to write ***equivalent inequalities***.

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$$5x + 4 < 4x + 2$$

$$5x + 4 + (-4) < 4x + 2 + (-4) \quad \text{Addition Prop. of Inequalities}$$

$$5x + (4 + (-4)) < 4x + (2 + (-4)) \quad \text{Associative Prop. of Addition}$$

$$5x + 0 < 4x + (-2) \quad \text{Additive Inverse \& Closure Props.}$$

$$5x < 4x - 2 \quad \text{Additive Identity \& the Defn. of Subtraction}$$

Example 1

Solve the inequality, $5x + 4 < 4x + 2$, then graph the solution.

Solution:

$$5x < 4x - 2$$

$$5x + (-4x) < 4x - 2 + (-4x) \quad \text{Addition Prop. of Inequalities}$$

$$5x + (-4x) < 4x + (-4x) - 2 \quad \text{Commutative Prop. of Addn.}$$

$$(5x + (-4x)) < (4x + (-4x)) - 2 \quad \text{Associative Prop. of Addn.}$$

$$(5 - 4) \cdot x < 0 - 2 \quad \text{Distributive \& Additive Inverse Props.}$$

$$1 \cdot x < -2 \quad \text{Closure \& Additive Identity Props.}$$

$$x < -2 \quad \text{Multiplicative Identity Prop.}$$

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Graph: We can shade the number line to the left of -2 to give a graphical description of the solution set.

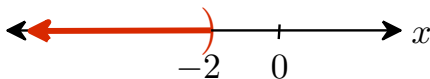


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We use a left-opening parenthesis at -2 to indicate that -2 is not part of the solution set.

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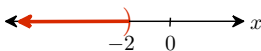
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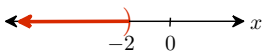
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Interval Notation

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Properties of Inequalities

Multiplication Property of Inequalities

For any three algebraic expressions A , B and C , where $C \neq 0$,

If $A < B$,

then $C \cdot A < C \cdot B$ if C is positive ($C > 0$)

or $C \cdot A > C \cdot B$ if C is negative ($C < 0$)

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$$\text{or } C \cdot A > C \cdot B \quad \text{if } C \text{ is negative } (C < 0)$$

In words: Multiplying both sides of an inequality by a positive quantity always produces an equivalent inequality. Multiplying both sides of an inequality by a negative number produces an equivalent inequality BUT it reverses the direction of the inequality symbol.

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Solution:

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$$-2x - 3 + 3 < 3 + 3 \quad \text{Addition Prop. of Inequalities}$$

$$-2x \leq 6 \quad \text{Additive Inverse \& Identity Props}$$

$$\left(-\frac{1}{2}\right) \cdot (-2x) \geq \left(-\frac{1}{2}\right) \cdot 6 \quad \text{Multiplication Prop. of Inequalities}$$

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Interval Notation

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Interval Notation and Graphing

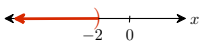
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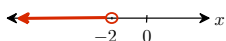
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Graph Using
Parenthesis/Brackets



Graph using open
and closed circles



Interval Notation and Graphing

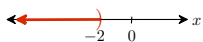
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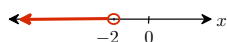
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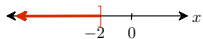


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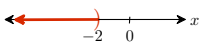
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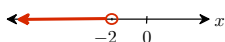
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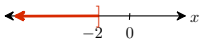


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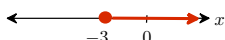
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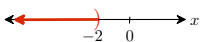
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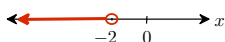
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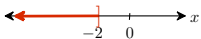


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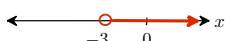
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Linear Inequalities in One Variable

Classroom Example: Solve the following inequality.

- $3(2x + 5) \leq -3x$

Linear Inequalities in One Variable

Classroom Examples: Take the next five minutes to work these 6 problems. Graph the solution set to the given inequality, then write the solution set using interval notation.

- $x \leq -6$
- $x > 5$
- $x \geq -1$
- $x > 10$

Classroom Examples: Solve each inequality. Graph the solution set, then write the solution set using interval notation.

- $2x - 1 \leq -6$
- $-3x < 2x - 6$

Linear Inequalities in One Variable

Definition

A compound inequality is two or more simple inequalities {sets} joined by the terms 'and' or 'or' .

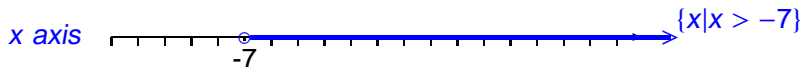
For Example, the set $\left\{ x \mid 3x - 6 \leq -3 \text{ or } 3x - 6 \geq 3 \right\}$ is a compound inequality.

The inequality statement $-7 < x < 7$ is to be read “x is in between -7 and 7 .”

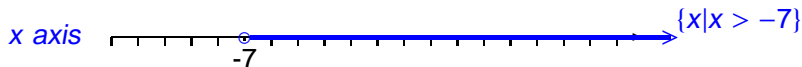
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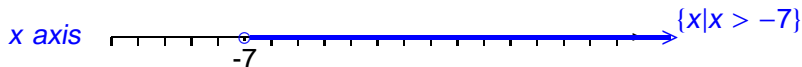
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Linear Inequalities in One Variable

Classroom Examples: Solve the following compound inequalities. Graph the solution set on a number line, then write the solution set using interval notation.

- $-7 \leq 2x + 1 \leq 7$

- $3x - 6 \leq -3$ or $3x - 6 \geq 3$

Interval Notation and Graphing

Inequality
Notation

$$-4 < x < 3$$

Interval
Notation

$$(-4, 3)$$

Graph Using
Parenthesis/Brackets



Graph using open
and closed circles



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$$-4 < x \leq 3$$

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Interval Notation and Graphing

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Equations with Absolute Value

Theorem

Suppose a and b are any real numbers with the restriction that $b > 0$. Then the equation $|a| = b$ is equivalent to $a = b$ or $a = -b$.

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Classroom Examples: Solve the following absolute value equations.

- $|x| = 4$
- $|3x - 6| = 9$
- $|4x - 3| + 2 = 3$