

## Lab 18: Section 13.2 Testing the Difference Between Means (Dependent Samples)

In Section 13.1, you performed two-sample hypothesis tests with *independent samples* using the test statistic  $\bar{x}_1 - \bar{x}_2$  (the difference between the means of the two samples). To perform a two-sample hypothesis test with *dependent samples*, you will use a different technique. You will first find the difference  $d$  for each data pair.

$$d = (\text{data entry in first sample}) - (\text{corresponding data entry in second sample})$$

The **test statistic** is the mean  $\bar{d}$  of these differences

$$\bar{d} = \frac{d}{n}$$

That is to say that  $\bar{d}$  is the average of the differences between paired data entries in the dependent samples

The **standardized test statistic** is

$$t = \frac{\bar{d} - \mu_d}{s_d/\sqrt{n}}$$

The degrees of freedom are

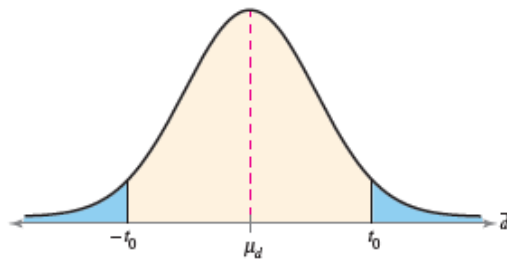
$$d.f. = n - 1.$$

These conditions are necessary to conduct the test.

1. The samples are randomly selected.
2. The samples are dependent (paired).
3. The populations are normally distributed or the number  $n$  of pairs of data is at least 30.

When these conditions are met, the sampling distribution for  $\bar{d}$ , the mean of the differences of the paired data entries in the dependent samples, is approximated by a t-distribution with  $n - 1$  degrees of freedom, where  $n$  is the number of data pairs.

***As a result, we can use the calculator's t-test function to get the p-value and the value of the standardized test statistic.***



The symbols listed in the table (below) are used for the t-test for  $\mu_d$ . Although formulas are given for the mean and standard deviation of differences, you should use technology to calculate these statistics.

Symbol	Description
$n$	The number of pairs of data
$d$	The difference between entries in a data pair
$\mu_d$	The hypothesized mean of the differences of paired data in the population
$\bar{d}$	The mean of the differences between the paired data entries in the dependent samples
	$\bar{d} = \frac{\sum d}{n}$
$s_d$	The standard deviation of the differences between the paired data entries in the dependent samples
	$s_d = \sqrt{\frac{\sum (d - \bar{d})^2}{n - 1}}$

**Classroom Exercises**

1. A medical researcher wants to determine whether a drug changes the body’s temperature. Seven test subjects are randomly selected, and the body temperature (in degrees Fahrenheit) of each is measured. The subjects are then given the drug and, after 20 minutes, the body temperature of each is measured again. The results are listed below. At  $\alpha = 0.05$ , is there enough evidence to conclude that the drug changes the body’s temperature? Assume the body temperatures are normally distributed.

Subject	1	2	3	4	5	6	7
Initial temperature	101.8	98.5	98.1	99.4	98.9	100.2	97.9
Second temperature	99.2	98.4	98.2	99.0	98.6	99.7	97.8

- (a) (2 points) Write the null and alternative hypotheses.
- (b) (2 points) What conditions should you check first before you conduct the hypothesis test?

- (c) (1 point) What formula should be used for the test statistic?
- (d) (1 point) What number is the test statistic equal to?
- (e) (1 point) What p-value do you obtain? Round to the ten-thousandths.
- (f) (1 point) Do you reject the null hypothesis or fail to reject the null hypothesis? Explain.
- (g) (2 points) Please write a conclusion sentence in the context of the problem.
2. **Pneumonia** A scientist claims that pneumonia causes weight loss in mice. The table below shows the weights (in grams) of six randomly selected mice before infection and two days after infection. At  $\alpha = 0.01$ , is there enough evidence to support the scientist's claim? Assume that weights are normally distributed.

Mouse	1	2	3	4	5	6
Weight (before)	19.8	20.6	20.3	22.1	23.4	23.6
Weight (after)	18.4	19.6	19.6	20.7	22.2	23.0

- (a) (2 points) Write the null and alternative hypotheses.
- (b) (2 points) What conditions should you check first before you conduct the hypoth-

esis test?

- (c) (1 point) What formula should be used for the test statistic?
- (d) (1 point) What number is the test statistic equal to?
- (e) (1 point) What p-value do you obtain? Round to the ten-thousandths.
- (f) (1 point) Do you reject the null hypothesis or fail to reject the null hypothesis? Explain.
- (g) (2 points) Please write a conclusion sentence in the context of the problem.