Lab 10: *The Binomial Distribution* Name: ____________

Use the following information to answer the first five exercises. Recently, a nurse in the small town of 'Supafly Rizzle Valley' commented that when a patient calls the medical advice line claiming to have covid-19, the chance that he or she truly has covid-19 (and not just a nasty cold) is only about 20% (see footnote).

Currently, the hospital where the nurse works is getting about 15 of these type of calls per day, and the local hospital has only 200 unoccupied hospital beds. Of the next 15 patients calling in claiming to have covid-19, we are interested in how many actually have covid-19.

footnote — 20% is roughly the Worldometer value from 3/30/20
total people tested: 944,854; total confirmed cases: 161,088

1. Define the random variable and list its possible values.

2. Find the probability that six of the next 15 calls are from people who actually have covid-19.

3. Find the probability that at least four of the 15 calls are from people who actually have covid-19.

$\geq$

4. On average, for every 15 people calling in, how many do you expect to actually have covid-19?

20% of 15 people $= (0.2)(15) = \boxed{3}$

p n

5. At this rate, in about how many days will the hospital be full?

We expect 3 patients a day to test

$200/3 \approx \boxed{67 \text{ days}}$

6. A student takes a ten-question true-false quiz, but did not study and randomly guesses each answer. Find the probability that the student passes the quiz with a grade of at least 70% of the questions correct.

① A trial is a test that determines if a person has covid-19

[Properties That define a binomial experiment

prop 1) There are a fixed number of trials (in a single day)

15 trials = 15 callers = 15 tests

prop 2) The trials are indep.

Since whether or not ~~the~~ ~~the~~ any one of the 15 has covid has no bearing on whether or not the other 14 have covid.

prop 3) There are 2 outcomes to each trial (test)

either the subject has covid-19 or they dont.

Success is defined "a subject is positive"

① The random variable x represents the number of successes in n trials.

X = the number of people (out of 15) testing positive

X	0	1	2	3	4	5	6 …	14	15

X can represent the numbers

$\boxed{0, 1, 2, 3, \ldots, 15}$ ← answer

② $P(6) = \boxed{0.0430}$

③ $P(x \geq 4)$

$$= \underbrace{P(4) + P(5) + P(6) + \ldots + P(15)}$$

$$= 1 - [P(0) + P(1) + P(2) + P(3)]$$

$$= 1 - 0.6481 = \boxed{0.3519}$$

20% of 15 people

For a binomial random varible

④ avg of $x = n \cdot p$

$$= 15(0.2) = \boxed{3}$$

3 callers per day are expected to have covid-19

(5) Assuming all 3 patients perday will need hospitalization,

$200/3 \doteq 66$ days

(6) prop1 There are a ~~fixed~~ finite number of trials

$n = 10$ trials ; = A test question response

prop2 The trials are indep.

Since whether or not any one question is correct has no bearing on whether any of the other questions are correct.

Success: A correct answer

prop3 2 outcomes: correct, not correct

prop.4 The prob. of success is the same (50%) for each trial (question) since there is a 50/50 chance at success on each question

6 $n = 10$ questions

$p = 0.50$

X = the number of correct answers out of 10 questions

x	0	1	2	3	4	5	6	7	8	9	10
$P(x)$	.	.	.	.	.	.	.	.	.	.	.

at least 70% = 7 or more correct answers

So, we need to find

$$\boxed{P(x \geq 7)}$$

6 $P(x \geq 7)$

$= P(7) + P(8) + P(9) + P(10)$

$= \boxed{0.1719}$

$n = 20$ trial = 20 yes/no questions

$p = 0.30$

(7) $X =$ the number of "yes" responses out of 20

(8) $0, 1, 2, 3, \ldots, 20$

(9) $P(x > 11) = P(12) + P(13) + \ldots + P(20)$

$= 0.0039 + 0.0010 + 0.0002$

$= \boxed{0.0051}$

(10) $P(x<3) = P(0)+P(1)+P(2)$

$= \boxed{0.0354}$

(11) If x is between 2 and 5, then x can only be 3 or 4, since x is a discrete random variable. So

$P(2<x<5)$

$= P(x=3 \text{ OR } x=4)$

$= P(3)+P(4)$

$= 0.0716+0.1304$

$= \boxed{0.202}$

(12) $P(6) = 0.1916$

(13) $P(3 \le x \le 5)$

$= P(3) + P(4) + P(5)$

$= \boxed{0.3809}$

(14) a $P(\text{none are prepared}) = P(0)$

$= \boxed{0.0008}$

$P(\text{all 20 are prepared}) = P(20)$

$= \boxed{0}$

It is more likely that none are prepared, since

$P(0) > P(20)$.

(15) avg = 30% of 20

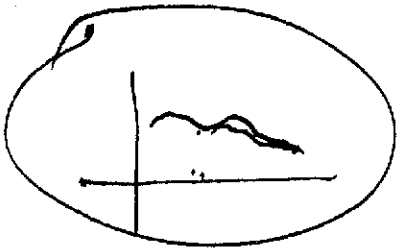
$= 0.3 \times 20$

$= \boxed{6 \text{ people}}$

$\mu = n \cdot p$

$= 20(0.3)$

$= 6$



Ch 6

Chapter 6
Probability Distns

Discrete Prob. Distns

Continuous Prob. Distns

X	0	12	13	14	15	16	. . .
$P(x)$	..	. -					

X is called a random variable
is used to represent the "number" out comes from a prob. experiment

X	1	2	3	4	5	6
$P(x)$	$1/6$	$1/6$	$1/6$	$1/6$	$1/6$	$1/6$

← Probability Distribution for the experiment: roll a six-sided die

X = the number the die lands on

Types of questions in Ch6

Prob. Distns

probability questions

mean, st dev, variance of the random variable

6.7 Binomial Distribution

The Binomial distn. is ~~an exp~~ a discrete prob. dist

The binomial RV. x is a discrete RV

Section 6.7

Professor Tim Busken

The Binomial Probability Distribution

Many probability experiments have only two outcomes. For example, when you guess at a multiple choice question, your answer is either right or wrong. A medical treatment can be considered effective or ineffective. Many survey questions can have only two possible answers: yes or no. When a coin is tossed it can land either heads or tails. Situations like these are called binomial experiments.

Binomial experiments are defined by the following properties:

1. The procedure has a fixed number of trials.
2. The trials must be independent. (The outcome of any individual trial doesn't affect the probabilities in the other trials.)
3. Each trial must have only two possible outcomes (commonly referred to as success and failure).
4. The probability of a success remains the same in all trials.

The outcomes of a binomial experiment and the corresponding probabilities of these outcomes are called a *binomial probability distribution*.

Notation for Binomial Probability Distributions:

n	denotes the fixed number of trials.
x	denotes a specific number of successes in n trials, so x can be any whole number between 0 and n, inclusive.
p	denotes the probability of success for a trial.
q	denotes the probability of failure for a trial, where $q = 1 - p$
$P(x)$ or $P(X = x)$	denotes the probability of getting exactly x successes among the n trials.

Here is an example of a binomial experiment: Pick a card from a standard deck and note whether or not the card is a club card. Then put the card back into the deck. Repeat the experiment five times, so $n = 5$. The outcomes of each trial can be classified in two categories. S = selecting a club and F = selecting another suit. The probability of success and failure are

$$p = \frac{1}{4} \quad \text{and} \quad q = 1 - \frac{1}{4} = \frac{3}{4}$$

The random variable x represents the number of clubs selected in five trials. So the possible values of the random variable are

$$0, 1, 2, 3, 4 \textit{ and } 5.$$

For instance, if $x = 2$, then exactly two of the five cards are clubs and the other three are not clubs.

Example of a binomial experiment

Suppose 58% of americans think ~~we are~~ that the US is in a recession or depre-

Suppose you take a random sample of 20 americans and ask them the yes/no question "Do you think the U.S is in a depression/recession"

This is a binomial experiment

4 Properties that define a binom. experiment

prop 1) fixed number of trials

A trial is a yes/no question

There are 20 trials

prop 2) Trials are independent

The responses to the survey questions are independent

prop 3) There are 2 outcomes per trial: "yes" or "no" property 4 The prob of success (58%) is the same for all $n = 20$ trials

58%: Success: person answered "yes"

42% failure: "no"

$n = 20$ questions (trials)

X = the number of people who said yes to the survey question.

$p = 0.58$

q = the prob of a "no" response

$q = 0.42$

X	0	1	2	3	...	20
$P(x)$		...				

0.000000029

These probabilities can be found with the binomial formula!

The probabilities are found using

$$P(x) = ({}_nC_x) \cdot p^x \cdot q^{n-x}$$

$$P(0) = ({}_{20}C_0) \cdot (0.58)^0 \cdot (0.42)^{20-0}$$

side work

$$({}_nC_x) = \frac{n!}{(n-x)!\,x!}$$

$\{1, 2, 3, \ldots\}$

! is called the factorial symbol

n! represents the product of the first n ~~whole~~ natural numbers

$3! = 3 \cdot 2 \cdot 1 = 6$

$4! = 4 \cdot 3 \cdot 2 \cdot 1 = 24$

$5! = 5 \cdot (4 \cdot 3 \cdot 2 \cdot 1) = 5 \cdot 4! = 120$

$0! = 1$ since we define it to be that way

$$ {}_{20}C_{0} = \frac{20!}{((20-0)!) \times 0!} $$

$$ = \frac{\cancel{20!}}{\cancel{20!} \times 1} = 1 $$

P(0) continued

$$ P(0) = 1 \cdot \cancel{(0.58)^{0}}^{1} \cdot (0.42)^{20} $$

$$ = (0.42)^{20} $$

$$ = 2.9 \times 10^{-8} $$

$$ = \boxed{0.000000029} $$

The prob 0 people out of 20 said yes is 0.000000029

Then,

$$P(1) = ({}_{20}C_1) \times (0.58)^1 \times (0.42)^{20-1}$$

$$\approx 8.1 \times 10^{-7} = \boxed{0.00000081}$$

$$P(2) = ({}_{20}C_2) \times (0.58)^2 \times (0.42)^{20-2}$$

$$\approx 1.1 \times 10^{-5} = \boxed{0.000011}$$

$$P(3) = ({}_{20}C_3)(0.58)^3 \times (0.42)^{20-3}$$

$$\approx 8.8 \times 10^{-5} = \boxed{0.000088}$$

So far, the binomial distribution table looks like this:

x	0	1	2	3
P(x)	0.000000029	0.00000081	0.000011	0.000088