Math 160 - Quiz 11

Due Thursday, May 9th

Name Key

Use the 5 step method of hypothesis testing to test the claims given below. Use both the Critical Value and P value methods.

1) Various temperature measurements are recorded at different times for a particular city. The mean of 20°C is obtained for 60 temperatures on 60 different days. Assuming that $\sigma = 1.5$°C, test the claim that the population mean is 22°C. Use a 0.05 significance level. 1) ______

2) The maximum acceptable level of a certain toxic chemical in vegetables has been set at 0.4 parts per million (ppm). A consumer health group measured the level of the chemical in a random sample of tomatoes obtained from one producer. The levels, in ppm, are shown below. 2) ______

0.31	0.47	0.19	0.72	0.56
0.91	0.29	0.83	0.49	0.28
0.31	0.46	0.25	0.34	0.17
0.58	0.19	0.26	0.47	0.81

Do the data provide sufficient evidence to support the claim that the mean level of the chemical in tomatoes from this producer is greater than the recommended level of 0.4 ppm? Use a 0.05 significance level to test the claim that these sample levels come from a population with a mean greater than 0.4 ppm. Use the P-value method of testing hypotheses. Assume that the standard deviation of levels of the chemical in all such tomatoes is 0.21 ppm.

3) The health of employees is monitored by periodically weighing them in. A sample of 54 employees has a mean weight of 183.9 lb. Assuming that σ is known to be 121.2 lb, use a 0.10 significance level to test the claim that the population mean of all such employees weights is less than 200 lb. 3) ______

Assume that a simple random sample has been selected from a normally distributed population. Find the test statistic, P-value, critical value(s), and state the final conclusion.

4) Test the claim that for the adult population of one town, the mean annual salary is given by $\mu = \$30,000$. Sample data are summarized as $n = 17$, $\bar{x} = \$22,298$, and $s = \$14,200$. Use a significance level of $\alpha = 0.05$. 4) ______

5) Test the claim that the mean lifetime of car engines of a particular type is greater than 220,000 miles. Sample data are summarized as $n = 23$, $\bar{x} = 226,450$ miles, and $s = 11,500$ miles. Use a significance level of $\alpha = 0.01$. 5) ______

6) Test the claim that the mean age of the prison population in one city is less than 26 years. Sample data are summarized as $n = 25$, $\bar{x} = 24.4$ years, and $s = 9.2$ years. Use a significance level of $\alpha = 0.05$. 6) ______

Use the traditional method to test the given hypothesis. Assume that the population is normally distributed and that the sample has been randomly selected.

7) The standard deviation of math test scores at one high school is 16.1. A teacher claims that the standard deviation of the girls' test scores is smaller than 16.1. A random sample of 22 girls results in scores with a standard deviation of 14.1. Use a significance level of 0.01 to test the teacher's claim. 7) ____________

8) A manufacturer uses a new production method to produce steel rods. A random sample of 17 steel rods resulted in lengths with a standard deviation of 2.1 cm. At the 0.10 significance level, test the claim that the new production method has lengths with a standard deviation different from 3.5 cm, which was the standard deviation for the old method. 8) ____________

9) In one town, monthly incomes for men with college degrees are found to have a standard deviation of $650. Use a 0.01 significance level to test the claim that for men without college degrees in that town, incomes have a higher standard deviation. A random sample of 22 men without college degrees resulted in incomes with a standard deviation of $971. 9) ____________

Use the traditional method to test the given hypothesis. Assume that the samples are independent and that they have been randomly selected

10) Use the given sample data to test the claim that $p_1 > p_2$. Use a significance level of 0.01. 10) ____________

Sample 1	Sample 2
$n_1 = 85$	$n_2 = 90$
$x_1 = 38$	$x_2 = 23$

11) Use the given sample data to test the claim that $p_1 < p_2$. Use a significance level of 0.10. 11) ____________

Sample 1	Sample 2
$n_1 = 462$	$n_2 = 380$
$x_1 = 84$	$x_2 = 95$

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

Construct the indicated confidence interval for the difference between population proportions $p_1 - p_2$. Assume that the samples are independent and that they have been randomly selected.

12) $x_1 = 30$, $n_1 = 66$ and $x_2 = 36$, $n_2 = 87$; Construct a 95% confidence interval for the difference between population proportions $p_1 - p_2$. 12) B

A) $-0.148 < p_1 - p_2 < 0.643$
B) $-0.118 < p_1 - p_2 < 0.199$
C) $0.296 < p_1 - p_2 < 0.613$
D) $0.266 < p_1 - p_2 < 0.643$

Construct the indicated confidence interval for the difference between the two population means. Assume that the two samples are independent simple random samples selected from normally distributed populations. Do not assume that the population standard deviations are equal.

13) Two types of flares are tested and their burning times are recorded. The summary statistics are given below. 13) D

Brand X	Brand Y
$n = 35$	$n = 40$
$\bar{x} = 19.4$ min	$\bar{x} = 15.1$ min
$s = 1.4$ min	$s = 0.8$ min

Construct a 95% confidence interval for the differences between the mean burning time of the brand X flare and the mean burning time of the brand Y flare.

A) $3.2 \text{ min} < \mu_X - \mu_Y < 5.4 \text{ min}$
B) $3.6 \text{ min} < \mu_X - \mu_Y < 5.0 \text{ min}$
C) $3.5 \text{ min} < \mu_X - \mu_Y < 5.1 \text{ min}$
D) $3.8 \text{ min} < \mu_X - \mu_Y < 4.8 \text{ min}$

Construct the indicated confidence interval for the difference between the two population means. Assume that the two samples are independent simple random samples selected from normally distributed populations. Also assume that the population standard deviations are equal ($\sigma_1 = \sigma_2$), so that the standard error of the difference between means is obtained by pooling the sample variances .

14) A researcher was interested in comparing the amount of time spent watching television by women and by men. Independent simple random samples of 14 women and 17 men were selected and each person was asked how many hours he or she had watched television during the previous week. The summary statistics are as follows. 14) D

Women	Men
$\bar{x}_1 = 12.6$ hr	$\bar{x}_2 = 16.5$ hr
$s_1 = 4.1$ hr	$s_2 = 4.7$ hr
$n_1 = 14$	$n_2 = 17$

Construct a 95% confidence interval for $\mu_1 - \mu_2$, the difference between the mean amount of time spent watching television for women and the mean amount of time spent watching television for men.

A) $-7.30 \text{ hrs} < \mu_1 - \mu_2 < -0.50 \text{ hrs}$
B) $-6.62 \text{ hrs} < \mu_1 - \mu_2 < -1.18 \text{ hrs}$
C) $-7.45 \text{ hrs} < \mu_1 - \mu_2 < -0.35 \text{ hrs}$
D) $-7.18 \text{ hrs} < \mu_1 - \mu_2 < -0.62 \text{ hrs}$

$-3.9 + ? = -0.62$

SHORT ANSWER. Write the word or phrase that best completes each statement or answers the question.

Perform the indicated hypothesis test. Assume that the two samples are independent simple random samples selected from normally distributed populations. Also assume that the population standard deviations are equal ($\sigma_1 = \sigma_2$), so that the standard error of the difference between means is obtained by pooling the sample variances.

15) A researcher was interested in comparing the response times of two different cab companies. Companies A and B were each called at 50 randomly selected times. The calls to company A were made independently of the calls to company B. The response times were recorded and the summary statistics were as follows: 15) ____________

	Company A	Company B
Mean response time	7.6 mins	6.9 mins
Standard deviation	1.4 mins	1.7 mins

Use a 0.02 significance level to test the claim that the mean response time for company A differs from the mean response time for company B. Use the P-value method of hypothesis testing.

16) A researcher was interested in comparing the resting pulse rates of people who exercise regularly and the pulse rates of those who do not exercise regularly. Independent simple random samples of 16 people who do not exercise regularly and 12 people who exercise regularly were selected, and the resting pulse rates (in beats per minute) were recorded. The summary statistics are as follows. 16) ____________

Do Not Exercise	Do Exercise
$\bar{x}_1 = 73.0$ beats/min	$\bar{x}_2 = 69.2$ beats/min
$s_1 = 10.1$ beats/min	$s_2 = 8.4$ beats/min
$n_1 = 16$	$n_2 = 12$

Use a 0.025 significance level to test the claim that the mean resting pulse rate of people who do not exercise regularly is greater than the mean resting pulse rate of people who exercise regularly. Use the traditional method of hypothesis testing.

① $n = 60$

$\bar{X} = 20°C$

$\sigma = 1.5°C$

$\alpha = 0.05$

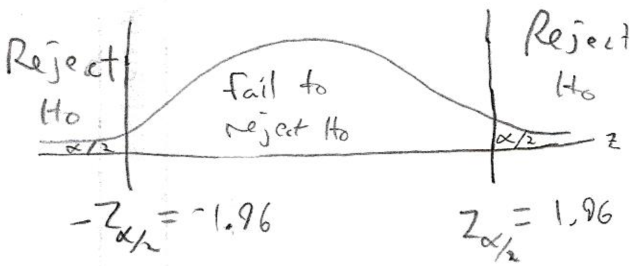
Claim $\mu = 22°$

Step 1 $H_0: \mu = 22$ (claim)

Step 2 $H_1: \mu \neq 22$ (2-tailed test)

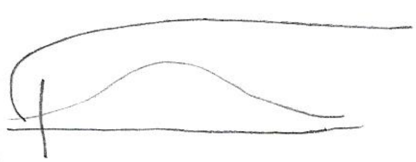
Step 3 the test statistic is $z = \dfrac{\bar{x} - \mu}{\sigma/\sqrt{n}} = \dfrac{20 - 22}{1.5/\sqrt{60}} \approx -10.33$

Step 4 $\boxed{\text{CV method}}$



The TS is located along the horizontal axis in the critical region, so reject Ho.

Step 4 $\boxed{\text{Pvalue method}}$



pvalue = twice this area

$\approx 2(0.0001) = 0.0002$

TS = −10.33

and p-val $\leq \alpha \Rightarrow$ reject H_0.

Step 5 There is sufficient evidence to warrant rejection of the claim that the population average temperature is 22° C.

(2) claim the mean level of chemical in tomatoes from this producer is greater than 0.4 ppm. $(\mu > 0.4)$

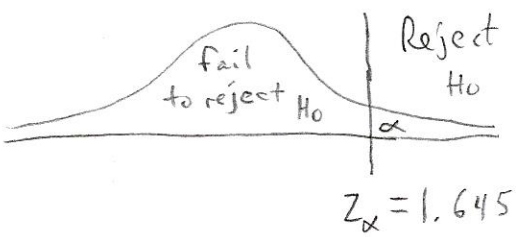
$\alpha = 0.05$, $\bar{x} = 0.4445$ ppm, $\sigma = 0.21$ ppm

Step 1 $H_0: \mu \le 0.4$

Step 2 $H_1: \mu > 0.4$ (claim, right-tailed test)

Step 3 The test statistic is $z = \dfrac{\bar{x} - \mu}{\sigma/\sqrt{n}} = \dfrac{0.4445 - 0.4}{0.21/\sqrt{20}} \approx 0.95$

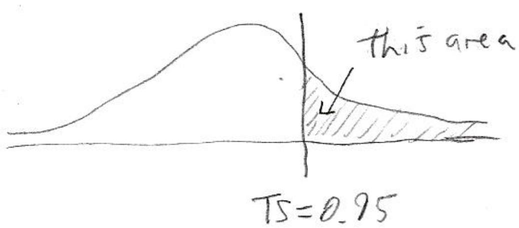
Step 4 $\boxed{\text{CV method}}$



The TS is located left of the CV, $z_\alpha = 1.645$, in the fail to reject region.

Step 4 $\boxed{\text{P value method}}$

p-value > α ⇒ fail to reject Ho



this area = p-value = $1 - 0.8289 = 0.1711$

Step 5 There is not sufficient sample evidence to support the claim that the level of chemical in tomatoes from this producer are greater than the recommended level of 0.4 ppm.

(3) $n = 54$

$\bar{x} = 183.9$ lb

$\sigma = 121.2$

$\alpha = 0.10$

Claim: The population mean of all employee weights is less than 200 lb.

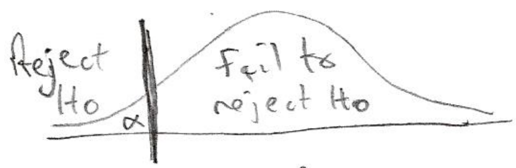
$\mu < 200$

Step 1 $H_0: \mu \geq 200$ lb.

Step 2 $H_1: \mu < 200$ lb. (claim, left-tailed test)

Step 3 The test statistic is $z = \dfrac{\bar{x} - \mu}{\sigma/\sqrt{n}} = \dfrac{183.9 - 200}{121.2/\sqrt{54}} = \dfrac{-16.1}{16.49323} \doteq -0.98$

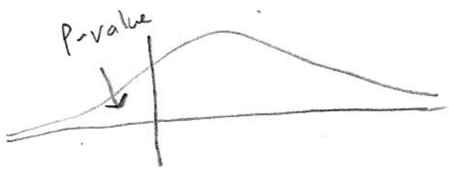
Step 4 $\boxed{\text{CV method}}$



$Z_\alpha = -1.28$

The TS is located right of the CV, $Z_\alpha = -1.28$, in the fail to reject region.

Step 4 $\boxed{\text{P-value Method}}$



TS $= -0.98$

P-value $\doteq 0.16 > \alpha \Rightarrow$ fail to reject H_0

Step 5 There is not suff. sample evidence to support the claim.

(4) Claim $\mu = \$30{,}000$ mean of adult pop. in town

$n = 17$

$\bar{x} = \$22{,}298$

$s = \$14{,}200$

$\alpha = 0.05$

Step 1 $H_0: \mu = 30{,}000$ (claim)

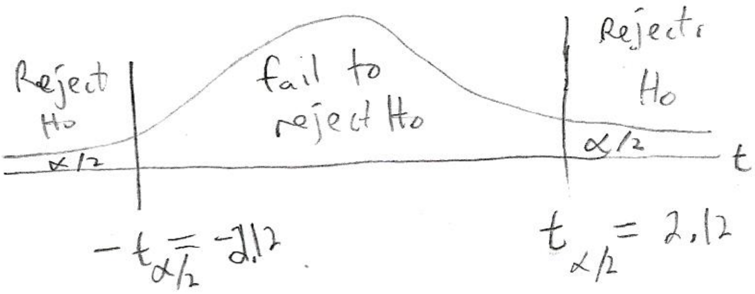
Step 2 $H_1: \mu \neq 30{,}000$ (2-tailed test)

Step 3 the test statistic is $t = \dfrac{\bar{x} - \mu}{s/\sqrt{n}} = \dfrac{22{,}298 - 30{,}000}{14{,}200/\sqrt{17}} \approx \dfrac{-7702}{3444.005}$

$\doteq \boxed{-2.24}$

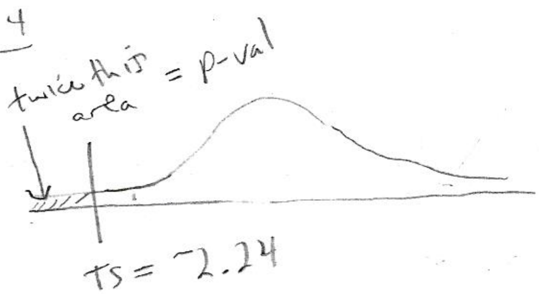
Step 4 $\boxed{\text{CV method}}$

$df = n - 1 = 16$



The TS lies in the critical region, so reject H_0.

Step 4



p-value $< 0.05 = \alpha \Rightarrow$ reject H_0

The area left of $-t_{\alpha/2} = -2.12$ is 0.025.

Since -2.12 is right of the TS, the p-value $< 2(0.025) = 0.05$.

TI-84 calculator p-value $= 0.0399$

Step 5 There is suff. sample evidence to reject the claim.

(5) Claim the mean lifetime of car engines of a particular type is greater than 220,000 miles. ($\mu > 220{,}000$)

$n = 23$,

$\bar{x} = 226{,}450$

$s = 11{,}500$

$\alpha = 0.01$

Step1 $H_0: \mu \leq 220{,}000$

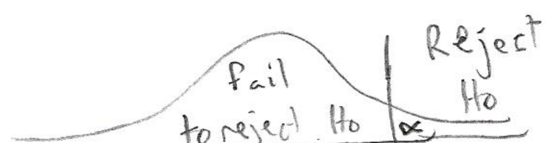
Step2 $H_1: \mu > 220{,}000$ (claim, right-tailed test)

Step3 The test statistic is $t = \dfrac{\bar{x} - \mu}{s/\sqrt{n}} = \dfrac{226{,}450 - 220{,}000}{11{,}500/\sqrt{23}} \approx \dfrac{6450}{2397.9157}$

≈ 2.69

Step4 $\boxed{\text{CV method}}$

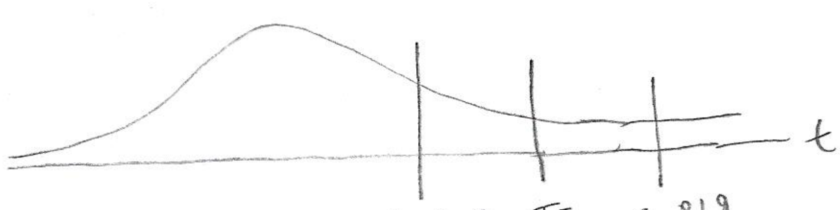
$df = n - 1 = 22$



$t_\alpha = 2.508$

The TS lies in the critical region. Reject Ho and accept the Claim.

Step4 $\boxed{\text{P-value method}}$



use row 22 of the t table

$0.005 < \text{p-val} < 0.01$

TI-84 calculator

p-value ≈ 0.00669

Step5 The sample data support the claim.

(6) claim: the mean age of the prison population in one city is less than 26 yrs. $(\mu < 26)$

$n = 25$ prisoners

$\bar{x} = 24.4$ yrs

$s = 9.2$ yrs

$\alpha = 0.05$

Step 1 $H_0: \mu \geq 26$

Step 2 $H_1: \mu < 26$ (claim, left-tailed test)

Step 3 the TS is $t = \dfrac{\bar{x} - \mu}{s/\sqrt{n}} = \dfrac{24.4 - 26}{9.2/\sqrt{25}} = \dfrac{-1.6}{1.84}$

$= \boxed{-0.87}$

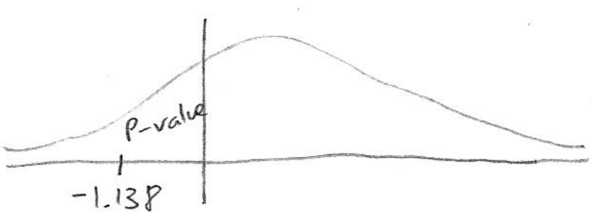
Step 4 CV method

$df = n - 1 = 24$

$t_\alpha = -1.711$

The TS does not lie in the critical region. Fail to reject Ho.

Step 4 P-value Method The p-value is the area under the sampling distribution that is left of the TS, $t = -0.87$.



TS = -0.87

From row 24 of the t table. The value $t = -1.318$ has associated with it an area to the left of 0.10. Since $t = -0.87$ is right of that, the p-value is greater than 0.10. Since p-val > α, fail to reject Ho. TI-84 calculator p-val = 0.1966.

Step 5 There is not sufficient sample evidence to support the claim that the mean age is less than 26.

(7) Claim: the standard deviation of girls test scores at one high school is less than 16.1 ($\sigma < 16.1$)

$n = 22$

$s = 14.1$

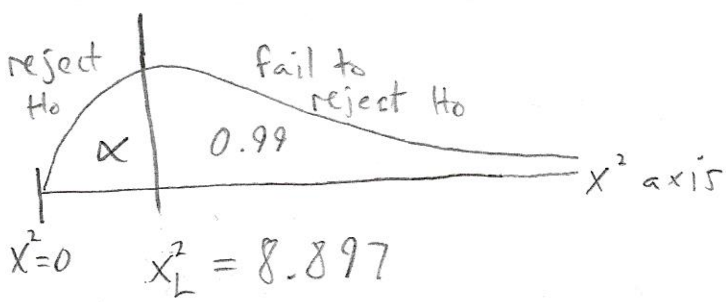
$\alpha = 0.01$

Step 1 $H_0 : \sigma \geq 16.1$

Step 2 $H_1 : \sigma < 16.1$ (claim, left-tailed test)

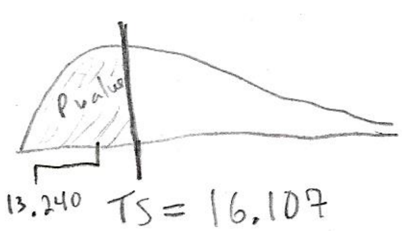
Step 3 The test statistic is $\chi^2 = \frac{(n-1)s^2}{\sigma^2} = \frac{(21)(14.1)^2}{16.1^2} \approx 16.107$

Step 4 $\boxed{\text{CV Method}}$ $df = n-1 = 21$



The TS is located along the horizontal axis, right of the CV, $\chi_L^2 = 8.897$, so it is not in the critical region. Fail to reject Ho.

Step 4 $\boxed{\text{Pvalue Method}}$ (not required)



From row 21 of the chi-square table, $\chi^2 = 13.240$ is the value closest to the TS, and $\chi^2 = 13.240$ has an area to the left equivalent to 0.10. Thus since $16.109 > 13.240$, the p-value is greater than 0.10. Hence, p-val $> \alpha \Rightarrow$ $\boxed{\text{fail to reject Ho}}$

Step 5 There is not sufficient sample evidence to support the claim.

8

$n = 17$

$s = 2.1$ cm

$\alpha = 0.10$

Claim: The new production method has lengths with a standard deviation that is different from (not equal to) 3.5 cm. ($\sigma \neq 3.5$ cm)

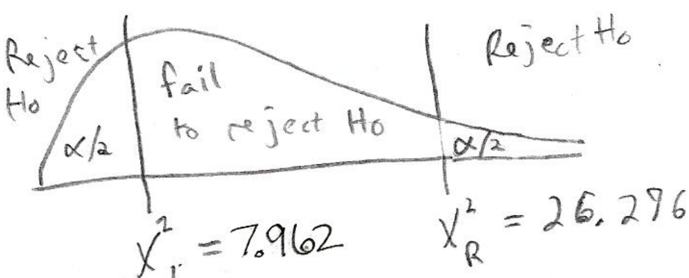
Step 1 $H_0 : \sigma = 3.5$ cm

Step 2 $H_1 : \sigma \neq 3.5$ cm (claim, 2-tailed test)

Step 3 The test statistic is $\chi^2 = \dfrac{(n-1)s^2}{\sigma^2} = \dfrac{16(2.1)^2}{(3.5)^2} = 5.76$

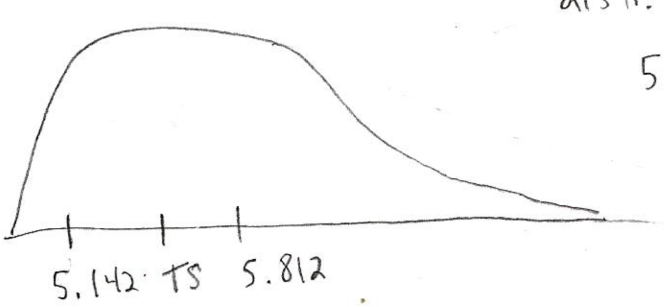
$df = n-1 = 16$

Step 4 $\boxed{\text{CV method}}$



The TS, $\chi^2 = 5.76$, is located in the critical region, so reject Ho.

Step 4 $\boxed{\text{P-value Method}}$ (not required) From row 16 of the chi-sq distr. table, since the TS is between 5.142 and 5.812, the p-value is between 0.005 and 0.01. Hence p-val < α, so reject Ho and support the claim.



Step 5 The sample data support the claim.

(9)

$\alpha = 0.01$
$n = 22$
$s = 971$

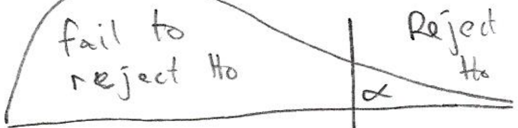
Claim: men w/o college degrees in 1 town have monthly incomes with standard deviation which is higher than \$650. ($\sigma > 650$)

Step 1 $H_0: \sigma \le 650$

Step 2 $H_1: \sigma > 650$ (claim, right-tailed test)

Step 3 The test statistic is $\chi^2 = \frac{(n-1)s^2}{\sigma^2} = \frac{21(971)^2}{650^2} = 46.863$

Step 4 $\boxed{\text{CV method}}$ $df = n-1 = 21$



$\chi^2_R = 38.932$

The TS lies in the critical region, reject Ho.

Step 4 $\boxed{\text{P-value}}$ (not required)

41.401

using row 21 of the chi sq table 41.401 is closest to the TS. 41.401 has an area to the left of 0.005. Since the TS is right of 41.401 (on the horizontal axis), the p-value is less than 0.005. Hence p-val $\le \alpha \Rightarrow$ reject Ho.

Step 5 The sample data support the claim.

TI-84 $\rightarrow$ Use 2-prop-Z-test

$n_1 = 85$	$n_2 = 90$
$x_1 = 38$	$x_2 = 23$
$\hat{p}_1 = x_1/n_1 = \frac{38}{85} \approx 0.4471$	$\hat{p}_2 = \frac{x_2}{n_2} = \frac{23}{90} \approx 0.2556$

10 step1 $H_0: \; p_1 - p_2 = 0$

Step2 $H_1: \; p_1 - p_2 > 0$ (claim, right-tailed test)

Step3

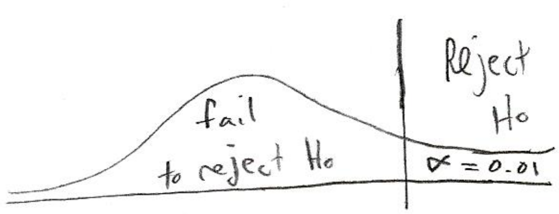
The test statistic is $z = \dfrac{(\hat{p}_1 - \hat{p}_2) - (p_1 - p_2)}{\sqrt{\dfrac{\bar{p}\bar{q}}{n_1} + \dfrac{\bar{p}\bar{q}}{n_2}}} = \dfrac{(0.4471 - 0.2556) - (0)}{\sqrt{\dfrac{(.3486)(.6514)}{85} + \dfrac{(.3486)(.6514)}{90}}}$

Where

$$\boxed{\begin{aligned}\bar{p} &= \frac{x_1 + x_2}{n_1 + n_2} = \frac{38 + 23}{85 + 90} = \frac{61}{175} \approx 0.3486 \\ \text{So, } \bar{q} &= 1 - \bar{p} = 1 - 0.3486 = 0.6514\end{aligned}}$$

$\approx \dfrac{0.1915}{0.0720735} \approx 2.66$

Step4 CV method



$z_\alpha = 2.33$

The TS, $z = 2.66$, is located along the horizontal axis, right of the critical value, $z_\alpha = 2.33$, in the critical region. Reject Ho. Support H_1.

Step 5 The sample data support the claim that $p_1 > p_2$.

Use a 2-prop-z-test

(11)

$x_1 = 84$ | $x_2 = 95$

$n_1 = 462$ | $n_2 = 380$

$\hat{p}_1 = \frac{84}{462} \approx 0.1818$ | $\hat{p}_2 = \frac{95}{380} = 0.25$

$\alpha = 0.10$

Claim $p_1 < p_2$ or $p_1 - p_2 < 0$

Step 1 $H_0 : (p_1 - p_2) = 0$

Step 2 $H_1 : (p_1 - p_2) < 0$ (claim, left tailed test)

Step 3 The test stat. is $z = \dfrac{(\hat{p}_1 - \hat{p}_2) - \cancel{(p_1 - p_2)}^{\,0}}{\sqrt{\dfrac{\bar{p}\bar{q}}{n_1} + \dfrac{\bar{p}\bar{q}}{n_2}}} = \dfrac{0.1818 - 0.25}{\sqrt{\dfrac{(.2126)(.7874)}{462} + \dfrac{(.2126)(.7874)}{380}}}$

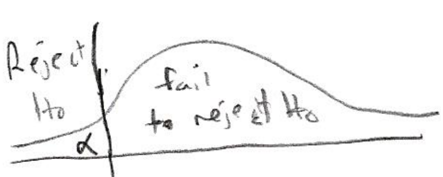
$\approx \dfrac{-0.0682}{0.0283349} \approx \boxed{-2.41}$

where $\bar{p} = \dfrac{x_1 + x_2}{n_1 + n_2}$

$= \dfrac{84 + 95}{462 + 380} = \dfrac{179}{842} \approx 0.2126$

and $\bar{q} = 1 - \bar{p} = 1 - 0.2126 = 0.7874$

Step 4 $\boxed{\text{CV method}}$



$z_\alpha = -1.28$

The TS lies in the critical region, so reject Ho.

Step 5 The sample data support the claim that $p_1 < p_2$.

(12) TI-84 use 2-prop-z-int, $\alpha = 1 - 0.95 = 0.05$

$$\hat{p}_1 = \frac{x_1}{n_1} = \frac{30}{66} \approx 0.4545 \; ; \; \hat{p}_2 = \frac{x_2}{n_2} = \frac{36}{87} \doteq 0.4138$$

use confidence interval formula:

$$(\hat{p}_1 - \hat{p}_2) - E < (p_1 - p_2) < (\hat{p}_1 - \hat{p}_2) + E$$

where $E = z_{\alpha/2} \cdot \sqrt{\frac{\hat{p}_1 \hat{q}_1}{n_1} + \frac{\hat{p}_2 \hat{q}_2}{n_2}} = 1.96 \sqrt{\frac{(.4545)(.5455)}{66} + \frac{(.4138)(.5862)}{87}} \approx 0.158562$

So, the estimate is

$$0.0407 - 0.1586 < p_1 - p_2 < 0.0407 + 0.1586$$

$$-0.1179 < p_1 - p_2 < 0.1993$$

B $\boxed{-0.118 < p_1 - p_2 < 0.199}$

(13) TI-84 Use 2-Samp-T-int pooled: no

Brand X — Brand Y

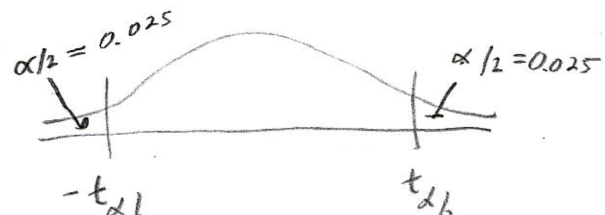
$n_1 = 35$, $\bar{x}_1 = 19.4$ min, $s_1 = 1.4$ min — $n_2 = 40$, $\bar{x}_2 = 15.1$ min, $s_2 = 0.8$

Confidence Interval Formula — $\alpha = 1 - 0.95 = 0.05$

$$(\bar{x}_1 - \bar{x}_2) - E < (\mu_1 - \mu_2) < (\bar{x}_1 - \bar{x}_2) + E$$

where $E = t_{\alpha/2}\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}$ and df = smaller of $n_1 - 1$ and $n_2 - 1$.

To find $t_{\alpha/2}$ we draw the sampling distribution. We locate $\pm t_{\alpha/2}$ on the horizontal axis then assign $\alpha/2 = 0.05/2 = 0.025$ to be the area in each tail. To find $t_{\alpha/2}$ on the t-table, we use row $df = n_1 - 1 = 34$, and area in 2 tails is 0.05 to get $t_{\alpha/2} = 2.032$.



Then $E = t_{\alpha/2}\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}} = (2.032)\sqrt{\frac{1.4^2}{35} + \frac{(0.8)^2}{40}} \doteq 0.5452$, and

$$(\bar{x}_1 - \bar{x}_2) - E < (\mu_1 - \mu_2) < (\bar{x}_1 - \bar{x}_2) + E$$

$$4.3 - 0.5452 < (\mu_1 - \mu_2) < 4.3 + 0.5452$$

$$3.7548 < (\mu_1 - \mu_2) < 4.8452$$

$$\boxed{3.8 \text{ min} < (\mu_1 - \mu_2) < 4.8 \text{ min}}$$

(14) TI-84 use 2-Samp-T-Int pooled: yes

$\alpha = 1 - 0.95 = 0.05$

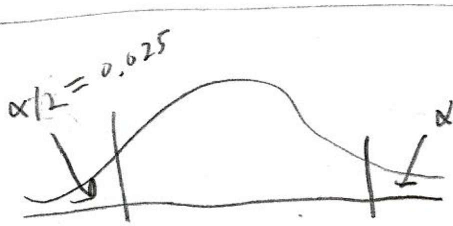
Confidence interval formula

$(\bar{x}_1 - \bar{x}_2) - E < (\mu_1 - \mu_2) < (\bar{x}_1 - \bar{x}_2) + E$, where

$E = t_{\alpha/2}\sqrt{\frac{s_p^2}{n_1} + \frac{s_p^2}{n_2}}$ and $df = n_1 + n_2 - 2$, with pooled
$= 13 + 16 - 2 = 27$

estimate of std dev. $s_p^2 = \frac{(n_1 - 1)s_1^2 + (n_2-1)s_2^2}{(n_1-1)+(n_2-1)} = \frac{(13)(4.1)^2 + 16(4.7)^2}{13+16}$

$\boxed{s_p^2 \approx 19.7231}$



$-t_{\alpha/2}$ $t_{\alpha/2} = 2.045$

df = 29 use row 29 of t-distn with area in 2 tails equal to $\alpha = 0.05$

$E = t_{\alpha/2}\sqrt{\frac{s_p^2}{n_1} + \frac{s_p^2}{n_2}} = (2.045)\sqrt{\frac{19.7231}{14} + \frac{19.7231}{17}} \doteq (2.045)(1.6028023)$
≈ 3.28

$(\bar{x}_1 - \bar{x}_2) - E < (\mu_1 - \mu_2) < (\bar{x}_1 - \bar{x}_2) + E$

$-3.9 - 3.28 < (\mu_1 - \mu_2) < -3.9 + 3.28$

$-7.19 < (\mu_1 - \mu_2) < -0.61$

TI-84 2 - Samp - T - test pooled: yes ✓

(15)

Company A	Company B
$\bar{X}_1 = 7.6$	$\bar{X}_2 = 6.9$
$s_1 = 1.4$	$s_2 = 1.7$
$n_1 = 50$	$n_2 = 50$

$\alpha = 0.02$

Claim: $\mu_1 - \mu_2 \neq 0$

Step 1 $H_0: \mu_1 - \mu_2 = 0$

Step 2 $H_1: \mu_1 - \mu_2 \neq 0$ (claim, 2-tailed test)

Step 3 The TS is $t = \dfrac{(\bar{X}_1 - \bar{X}_2) - (\mu_1 - \mu_2)}{\sqrt{\dfrac{s_p^2}{n_1} + \dfrac{s_p^2}{n_2}}} = \dfrac{(7.6 - 6.9) - 0}{\sqrt{\dfrac{2.425}{50} + \dfrac{2.425}{50}}}$

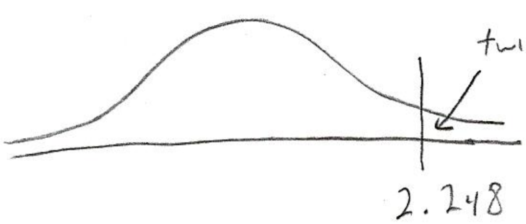
$$s_p^2 = \frac{(n_1 - 1)\, s_1^2 + (n_2 - 1)\, s_2^2}{n_1 + n_2 - 2}$$

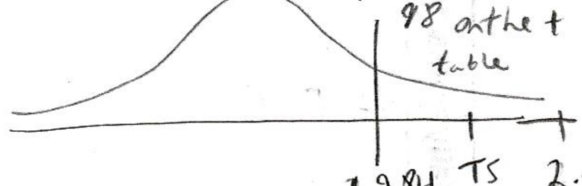
$$= \frac{49(1.4)^2 + 49(1.7)^2}{50 + 50 - 2} = \boxed{2.425}$$

$$\approx \frac{0.7}{0.3114483} \doteq 2.248$$

Step 4 $\boxed{\text{P-value}}$

$df = 98$ we can use $df = 100$ since there is no row 98 on the t table





From row 100 of the t-table, since the TS is between 1.984 & 2.364, the p-value is between 2(0.025) and 2(0.01), or equivalently, the p-value is between 0.05 and 0.02. This is greater than α.

Step 5 There is not sufficient sample evidence to support the claim

(16)

Do not exercise

$\bar{X}_1 = 73.$

$s_1 = 10.1$

$n_1 = 16$

Do exercise

$\bar{X}_2 = 69.2$

$s_2 = 8.4$

$n_2 = 12$

$\alpha = 0.025$

claim $\mu_1 > \mu_2$ or

$\mu_1 - \mu_2 > 0$

Use TI-84
2-Samp-T-test pooled: yes

Step 1 $H_0: \mu_1 - \mu_2 = 0$

Step 2 $H_1: \mu_1 - \mu_2 > 0$ (claim, right-tailed test)

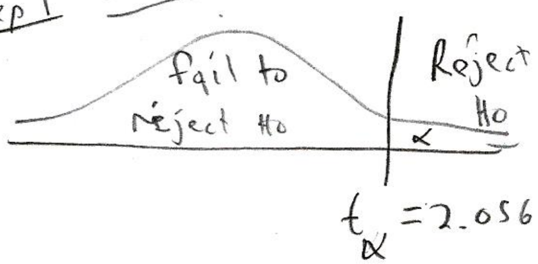
Step 3 The TS. is $t = \dfrac{(\bar{x}_1 - \bar{x}_2) - (\mu_1 - \mu_2)}{\sqrt{\dfrac{s_p^2}{n_1} + \dfrac{s_p^2}{n_2}}} = \dfrac{(73 - 69.2) - 0}{\sqrt{\dfrac{88.7042}{16} + \dfrac{88.7042}{12}}}$

$$s_p^2 = \frac{(n_1-1)s_1^2 + (n_2-1)s_2^2}{n_1 + n_2 - 2} = \frac{15(10.1)^2 + 11(8.4)^2}{16 + 12 - 2}$$

$\doteq 88.70423$

$\approx \dfrac{3.8}{3.596669} \doteq 1.057$

Step 4 CV method



df = 26

The TS is located in the fail to reject Ho region.

Step 5 There is not suff. sample evidence to support the claim.

Answer Key
Testname: UNTITLED1

Quiz 11 Key

1) H_0: $\mu = 22$; H_1: $\mu \neq 22$. Test statistic: $z = -10.33$. P–value: 0.0002. Because the P–value is less than the significance level of $\alpha = 0.05$, we reject the null hypothesis. There is sufficient evidence to warrant rejection of the claim that the population mean temperature is 22°C.

2) H_0: $\mu = 0.4$ ppm

H_1: $\mu > 0.4$ ppm

Test statistic: $z = 0.95$

P–value: 0.1711

Do not reject H_0; At the 5% significance level, the data do not provide sufficient evidence to support the claim that the mean level of the chemical in tomatoes from this producer is greater than the recommended level of 0.4 ppm.

3) H_0: $\mu = 200$; H_1: $\mu < 200$; Test statistic: $z = -0.98$. P–value: 0.1635. Fail to reject H_0. There is not sufficient evidence to support the claim that the mean is less than 200 pounds.

4) $\alpha = 0.05$

Test statistic: $t = -2.236$

P–value: $p = 0.0399$

Critical values: $t = \pm 2.120$

Because the test statistic, $t < -2.120$, we reject the null hypothesis. There is sufficient evidence to warrant rejection of the claim that $\mu = \$30,000$.

5) $\alpha = 0.01$

Test statistic: $t = 2.6898$

P–value: $p = 0.0066$

Critical value: $t = 2.508$

Because the test statistic, $t > 2.508$, we reject the null hypothesis. There is sufficient evidence to accept the claim that $\mu > 220,000$ miles.

6) $\alpha = 0.05$

Test statistic: $t = -0.87$

P–value: $p = 0.1966$

Critical value: $t = -1.711$

Because the test statistic, $t > -1.711$, we do not reject the null hypothesis. There is not sufficient evidence to support the claim that the mean age is less than 26 years.

7) Test statistic: $\chi^2 = 16.107$. Critical value: $\chi^2 = 8.897$. Fail to reject H_0. There is not sufficient evidence to support the claim that the standard deviation of the girls' test scores is smaller than 16.1.

8) Test statistic: $\chi^2 = 5.760$. Critical values: $\chi^2 = 7.962, 26.296$. Reject H_0. There is sufficient evidence to support the claim that the standard deviation is different from 3.5.

9) Test statistic: $\chi^2 = 46.863$. Critical values: $\chi^2 = 38.932$. Reject H_0. There is sufficient evidence to support the claim that incomes of men without college degrees have a standard deviation greater than $650.

10) H_0: $p_1 = p_2$. H_1: $p_1 > p_2$.

Test statistic: $z = 2.66$. Critical value: $z = 2.33$.

Reject the null hypothesis. There is sufficient evidence to support the claim that $p_1 > p_2$.

11) H_0: $p_1 = p_2$. H_1: $p_1 < p_2$.

Test statistic: $z = -2.41$. Critical value: $z = -1.28$.

Reject the null hypothesis. There is sufficient evidence to support the claim that $p_1 < p_2$.

12) B

13) D

14) D

15) H_0: $\mu_1 = \mu_2$
H_1: $\mu_1 \neq \mu_2$
Test statistic: $t = 2.248$
$0.02 < \text{P-value} < 0.05$
Do not reject H_0. At the 2% significance level, there is not sufficient evidence to support the claim that the mean response time for company A differs from the mean response time for company B.

16) H_0: $\mu_1 = \mu_2$
H_1: $\mu_1 > \mu_2$
Test statistic: $t = 1.057$
Critical value: $t = 2.056$
Do not reject H_0. At the 2.5% significance level, there is not sufficient evidence to support the claim that the mean resting pulse rate of people who do not exercise regularly is greater than the mean resting pulse rate of people who exercise regularly.