

# Math 160

## Binomial Distribution Quiz 7

Due Tuesday April 9th

Name: \_\_\_\_\_

Key

**Multiple Choice Section.** For questions, 1 through 6, choose the one alternative that best completes the statement or answers the question.

1. A machine has 12 identical components which function independently. The probability that a component will fail is 0.2. The machine will stop working if more than three components fail. Find the probability that the machine will be working.

A) 0.795 B) 0.927 C) 0.133 D) 0.206

1. A

2. A company purchases shipments of machine components and uses this acceptance sampling plan: Randomly select and test 30 components and accept the whole batch if there are fewer than 3 defectives. If a particular shipment of thousands of components actually has a 6% rate of defects, what is the probability that this whole shipment will be accepted?

A) 0.576 B) 0.277 C) 0.732 D) 0.165

2. C

3. A car insurance company has determined that 9% of all drivers were involved in a car accident last year. Among the 11 drivers living on one particular street, 3 were involved in a car accident last year. If 11 drivers are randomly selected, what is the probability of getting 3 or more who were involved in a car accident last year?

A) 0.057 B) 0.943 C) 0.424 D) 0.070

3. D

4. An airline estimates that 90% of people booked on their flights actually show up. If the airline books 71 people on a flight for which the maximum number is 69, what is the probability that the number of people who show up will exceed the capacity of the plane?

A) 0.022 B) 0.001 C) 0.004 D) 0.005

4. D

5. According to a college survey, 22% of all students work full time. Find the mean for the number of students who work full time in samples of size 16.

A) 3.5 B) 2.8 C) 0.2 D) 4.0

5. A

6. According to a college survey, 22% of all students work full time. Find the standard deviation for the number of students who work full time in samples of size 16.

A) 1.7 B) 3.5 C) 2.6 D) 1.9

6. A

$$n = 12$$

$$p = 0.2$$

$$P(X \leq 3)$$

### Binomial Experiment

People with type O-negative blood are said to be "universal donors." About 7% of the U.S. population has this blood type. Suppose that 30 people show up at a blood drive. Let  $x$  = the number of universal donors among a random group of 30 people.

- $n$  This is the number of trials. For this example,  $n = 30$  (the number of blood donors).
- $p$  This is the "success" probability. For this example,  $p = 0.07$  (the probability that a randomly selected American has type O-negative blood). Note that  $p$  must be in decimal form.
- $x$  This is the number of "successes," or type-O negative donors

$$\mu = n \cdot p = 30(0.07) = 2.1 ; \quad \sigma = \sqrt{npq} = \sqrt{(30)(0.07)(0.93)} \approx 1.40$$

7. Using the range rule of thumb, what is range of usual  $x$  values for this blood drive?

$$X_{\min} = \mu - 2\sigma = 2.1 - 2(1.4) = -0.7$$

7. [0, 4] It is usual to expect between 0 and 4

$$X_{\max} = \mu + 2\sigma = 2.1 + 2(1.4) = 4.9$$

8. Find the probability that **none** of the people who show up are type-O negative.

$$\text{Find } P(x=0) = \text{binompdf}(n, p, x) = \text{binompdf}(30, 0.07, 0) = \underline{0.1134}$$

universal donors to show up.

9. Find the probability that **exactly** 4 people show up who are type-O negative.

$$P(x=4) = \text{binompdf}(n, p, x) = \text{binompdf}(30, 0.07, 4) = \underline{0.0997}$$

10. Find the probability,  $P(x \leq 4)$ .

$$P(x \leq 4) = P(x=0) + P(x=1) + P(x=2) + P(x=3) + P(x=4) = \text{binomcdf}(30, 0.07, 4) = \underline{0.9447}$$

11. Find the probability,  $P(x = 4)$ .

$$P(x=4) = 0.0997$$

11. 0.0997

Find the probability that the number of type-O negative donors who show up:

12. **will not exceed 5**

$$P(x \leq 5) = \text{binomcdf}(30, 0.07, 5) = \underline{0.9838}$$

13. **is between 4 and 6.**

$$P(4 < x < 6) = P(x=5)$$

13. 0.0390

$$= \text{binompdf}(30, 0.07, 5)$$

## Quiz 7 Key

①

$n = 12$  trials (machine components)

$p = 0.2$  = the probability that a single component will fail.

Let  $X$  = the discrete random variable (DRV) representing the number of components that are failing (broken).

Then, the question asks us to find  $P(X \leq 3)$  since the machine will work with 0 parts failing, or 1 part failing, or 2 broken components, or 3 broken components.

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The criteria of a binomial probability distribution is met.

$$P(X \leq 3) = P(X=0) + P(X=1) + P(X=2) + P(X=3)$$

$$= \text{binomcdf}(n, p, 3)$$

$$= \text{binomcdf}(12, 0.2, 3)$$

$$\approx 0.795$$



(2)

$n = 30$  components

$p = 0.06$  defective rate

Let  $x =$  the DRV representing the number of defectives in a sample of 30.

Find  $P(x < 3)$ .

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$$P(x < 3) = P(x \leq 2) = P(x=0) + P(x=1) + P(x=2)$$

$$= \text{binomcdf}(n, p, 2)$$

$$= \text{binomcdf}(30, 0.06, 2) \approx 0.732$$

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(3)

$p = 9\% = 0.09$  = the probability a person gets in a car accident in a years time.

$n = 11$  people, let  $x =$  the number of people involved in a car accident in a single yr.

$$\text{Then } P(x \geq 3) = P(x=3) + P(x=4) + \dots + P(x=11)$$

$$= 1 - P(x < 3) = 1 - P(x \leq 2)$$

$$= 1 - \text{binomcdf}(n, p, 2) \approx 0.07$$



④

$n = 71$  trials (bookings)

$p = 0.90$  = the probability a person who booked the flight actually shows up.

Let  $x$  = the number of people who actually show up.

Find  $P(x > 69)$ .

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$$P(x > 69) = P(x = 70 \text{ or } x = 71)$$

$$= P(x = 70) + P(x = 71)$$

$$= \text{binompdf}(n, p, 70) + \text{binompdf}(n, p, 71)$$

$$\approx 0.005$$

⑤

$$n = 16$$

$$p = 0.22$$

$$q = 1 - 0.22 = 0.88$$

} use the binom dist.

$$\mu = n \cdot p = (16) \cdot (0.22) \approx 3.52$$

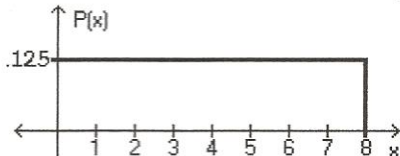
$$\sigma = \sqrt{npq} = \sqrt{16(0.22)(0.88)} \approx 1.76$$

⑥

Key

**MULTIPLE CHOICE.** Choose the one alternative that best completes the statement or answers the question.

Using the following uniform density curve, answer the question.



1) What is the probability that the random variable has a value greater than 4?

A) 0.450

B) 0.625

C) 0.375

D) 0.500

1) D

Assume that the weight loss for the first month of a diet program varies between 6 pounds and 12 pounds, and is spread evenly over the range of possibilities, so that there is a uniform distribution. Find the probability of the given range of pounds lost.

2) Between 8 pounds and 11 pounds

A)  $\frac{1}{3}$

B)  $\frac{2}{3}$

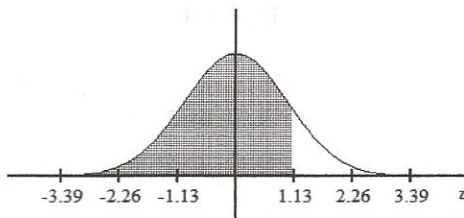
C)  $\frac{1}{2}$

D)  $\frac{1}{4}$

2) C

Find the area of the shaded region. The graph depicts the standard normal distribution with mean 0 and standard deviation 1.

3)



A) 0.1292

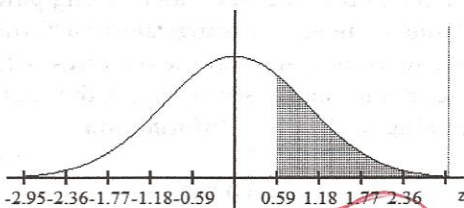
B) 0.8485

C) 0.8708

D) 0.8907

3) C

4)



A) 0.7224

B) 0.2776

C) 0.2190

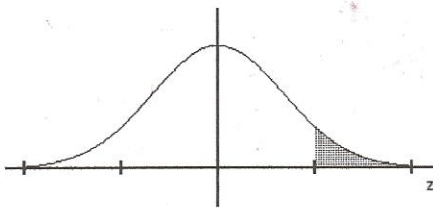
D) 0.2224

4) B

Find the indicated z score. The graph depicts the standard normal distribution with mean 0 and standard deviation 1.

5) Shaded area is 0.0694.

5) D



A) 1.39

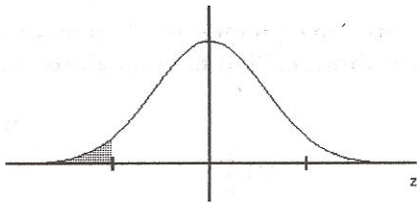
B) 1.45

C) 1.26

D) 1.48

6) Shaded area is 0.0401.

6) C



A) -1.89

B) -1.48

C) -1.75

D) -1.63

If z is a standard normal variable, find the probability.

7) The probability that z lies between -2.41 and 0

7) B

A) 0.0948

B) 0.4920

C) 0.5080

D) 0.4910

8) The probability that z lies between 0 and 3.01

8) D

A) 0.1217

B) 0.5013

C) 0.9987

D) 0.4987

9)  $P(-0.73 < z < 2.27)$

9) D

A) 1.54

B) 0.2211

C) 0.4884

D) 0.7557

The Precision Scientific Instrument Company manufactures thermometers that are supposed to give readings of  $0^{\circ}\text{C}$  at the freezing point of water. Tests on a large sample of these thermometers reveal that at the freezing point of water, some give readings below  $0^{\circ}\text{C}$  (denoted by negative numbers) and some give readings above  $0^{\circ}\text{C}$  (denoted by positive numbers). Assume that the mean reading is  $0^{\circ}\text{C}$  and the standard deviation of the readings is  $1.00^{\circ}\text{C}$ . Also assume that the frequency distribution of errors closely resembles the normal distribution. A thermometer is randomly selected and tested. Find the temperature reading corresponding to the given information.

10) Find  $Q_3$ , the third quartile.

10) A

A)  $0.67^{\circ}$

B)  $0.82^{\circ}$

C)  $-1.3^{\circ}$

D)  $0.53^{\circ}$

11) If 7% of the thermometers are rejected because they have readings that are too high, but all other thermometers are acceptable, find the temperature that separates the rejected thermometers from the others.

11) C

A)  $1.45^{\circ}$

B)  $1.39^{\circ}$

C)  $1.48^{\circ}$

D)  $1.26^{\circ}$

Find the indicated value.

12)  $z_{0.36}$

12) B

A) 1.60

B) 0.36

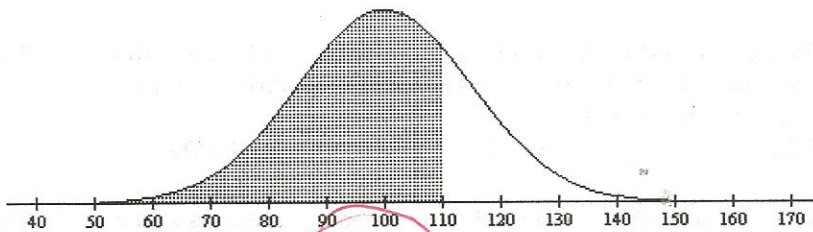
C) 0.45

D) 1.76



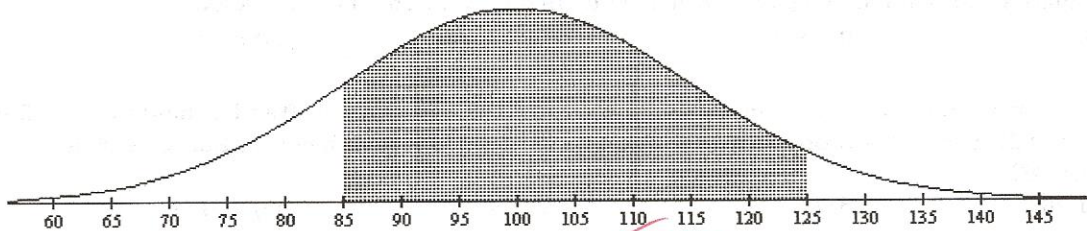
Provide an appropriate response.

- 13) Find the area of the shaded region. The graph depicts IQ scores of adults, and those scores are normally distributed with a mean of 100 and a standard deviation of 15 (as on the Wechsler test). 13) B



- A) 0.6293      B) 0.7486      C) 0.4400      D) 0.8051

- 14) Find the area of the shaded region. The graph depicts IQ scores of adults, and those scores are normally distributed with a mean of 100 and a standard deviation of 15 (as on the Wechsler test). 14) C



- A) 0.7745      B) 0.7619      C) 0.7938      D) 0.7303

- 15) Find  $P_{40}$ , which is the IQ score separating the bottom 40% from the top 60%.

- A) 96.6      B) 95.6      C) 96.1      D) 95.2

A or C works

- 15) A or C

Solve the problem. Round to the nearest tenth unless indicated otherwise.

- 16) Suppose that replacement times for washing machines are normally distributed with a mean of 9.4 years and a standard deviation of 2 years. Find the replacement time that separates the top 18% from the bottom 82%.

- A) 11.2 years      B) 7.6 years      C) 9.8 years      D) 10.6 years

- 16) A

- 17) The weights of certain machine components are normally distributed with a mean of 8.2 g and a standard deviation of 0.1 g. Find the two weights that separate the top 3% and the bottom 3%. These weights could serve as limits used to identify which components should be rejected. Round to the nearest hundredth of a gram.

- A) 8.01 g and 8.39 g      B) 7.98 g and 8.47 g  
C) 8.18 g and 8.22 g      D) 8.15 g and 8.25 g

- 17) A

Find the indicated probability.

- 18) The diameters of bolts produced by a certain machine are normally distributed with a mean of 0.30 inches and a standard deviation of 0.01 inches. What percentage of bolts will have a diameter greater than 0.32 inches? 18) B

- A) 97.72%      B) 2.28%      C) 37.45%      D) 47.72%



- 19) The diameters of pencils produced by a certain machine are normally distributed with a mean of 0.30 inches and a standard deviation of 0.01 inches. What is the probability that the diameter of a randomly selected pencil will be less than 0.285 inches?

A) 0.4332      B) 0.0668      C) 0.9332      D) 0.0596

19) B

- 20) A bank's loan officer rates applicants for credit. The ratings are normally distributed with a mean of 200 and a standard deviation of 50. If an applicant is randomly selected, find the probability of a rating that is between 170 and 220.

A) 0.3811      B) 0.2257      C) 0.1554      D) 0.0703

20) A

- 21) The lengths of human pregnancies are normally distributed with a mean of 268 days and a standard deviation of 15 days. What is the probability that a pregnancy lasts at least 300 days?

A) 0.0179      B) 0.0166      C) 0.4834      D) 0.9834

21) B

**Solve the problem.**

- 22) Scores on a test have a mean of 73 and  $Q_3$  is 81. The scores have a distribution that is approximately normal. Find  $P_{90}$ . (You will need to first find the standard deviation.)

A) 87.1      B) 88.8      C) 87.6      D) 88.3

22) D

- 23) The scores on a certain test are normally distributed with a mean score of 60 and a standard deviation of 2. What is the probability that a sample of 90 students will have a mean score of at least 60.2108?

A) 0.1587      B) 0.8413      C) 0.3413      D) 0.3174

23) A

- 24) For women aged 18–24, systolic blood pressures (in mm Hg) are normally distributed with a mean of 114.8 and a standard deviation of 13.1. If 23 women aged 18–24 are randomly selected, find the probability that their mean systolic blood pressure is between 119 and 122.

A) 0.0577      B) 0.0833      C) 0.9341      D) 0.3343

24) A

- 25) Human body temperatures are normally distributed with a mean of 98.20°F and a standard deviation of 0.62°F. If 19 people are randomly selected, find the probability that their mean body temperature will be less than 98.50°F.

A) 0.9826      B) 0.4826      C) 0.3343      D) 0.0833

25) A

- 26) Suppose that replacement times for washing machines are normally distributed with a mean of 9.3 years and a standard deviation of 1.1 years. Find the probability that 70 randomly selected washing machines will have a mean replacement time less than 9.1 years.

A) 0.4286      B) 0.0714      C) 0.0643      D) 0.4357

26) C

- 27) A study of the amount of time it takes a mechanic to rebuild the transmission for a 2005 Chevrolet Cavalier shows that the mean is 8.4 hours and the standard deviation is 1.8 hours. If 40 mechanics are randomly selected, find the probability that their mean rebuild time exceeds 8.7 hours.

A) 0.1346      B) 0.1469      C) 0.1946      D) 0.1285

27) B

- 28) A final exam in Math 160 has a mean of 73 with standard deviation 7.8. If 24 students are randomly selected, find the probability that the mean of their test scores is greater than 78.

A) 0.0103      B) 0.0036      C) 0.0008      D) 0.8962

28) C

*Key***SHORT ANSWER.** Write the word or phrase that best completes each statement or answers the question.

Determine whether the following is a probability distribution. If not, identify the requirement that is not satisfied.

1)

x	P(x)
0	0.079
1	0.173
2	-0.030
3	0.170
4	0.075
5	0.533

1) no*Not a probability distribution.**← One of the probabilities is negative*

2) A police department reports that the probabilities that 0, 1, 2, 3, and 4 car thefts will be reported in a given day are 0.135, 0.271, 0.271, 0.180, and 0.090, respectively.

2) no*The sum of the probabilities is not 1.***MULTIPLE CHOICE.** Choose the one alternative that best completes the statement or answers the question.**Find the mean of the given probability distribution.**

3) The number of golf balls ordered by customers of a pro shop has the following probability distribution.

3) B

x	P(x)
3	0.14
6	0.25
9	0.36
12	0.15
15	0.10

A)  $\mu = 5.79$ B)  $\mu = 8.46$ C)  $\mu = 9.06$ D)  $\mu = 9$ **Provide an appropriate response. Round to the nearest hundredth.**

4) Find the standard deviation for the given probability distribution.

4) C

x	P(x)
0	0.12
1	0.17
2	0.09
3	0.28
4	0.34

A)  $\sigma = 1.99$ B)  $\sigma = 2.91$ C)  $\sigma = 1.41$ D)  $\sigma = 1.45$ **Answer the question.**

5) Assume that there is a 0.05 probability that a sports playoff series will last four games, a 0.45 probability that it will last five games, a 0.45 probability that it will last six games, and a 0.05 probability that it will last seven games. Is it unusual for a team to win a series in 4 games?

5) A

A) Yes

B) No



- 6) Suppose that computer literacy among people ages 40 and older is being studied and that the accompanying table describes the probability distribution for four randomly selected people, where  $x$  is the number that are computer literate. Is it unusual to find four computer literates among four randomly selected people?

$x$	$P(x)$
0	0.16
1	0.25
2	0.36
3	0.15
4	0.08

A) Yes

B) No

6) B

key

Assume that a researcher randomly selects 14 newborn babies and counts the number of girls selected,  $x$ . The probabilities corresponding to the 14 possible values of  $x$  are summarized in the given table. Answer the question using the table.

Probabilities of Girls

$x(\text{girls})$	$P(x)$	$x(\text{girls})$	$P(x)$	$x(\text{girls})$	$P(x)$
0	0.000	5	0.122	10	0.061
1	0.001	6	0.183	11	0.022
2	0.006	7	0.209	12	0.006
3	0.022	8	0.183	13	0.001
4	0.061	9	0.122	14	0.000

- 7) Find the probability of selecting exactly 8 girls.

A) 0.000

B) 0.183

C) 0.122

D) 0.022

7) B

- 8) Find the probability of selecting 9 or more girls.

A) 0.122

B) 0.001

C) 0.212

D) 0.061

8) C

- 9) Find the probability of selecting exactly 4 girls.

A) 0.122

B) 0.001

C) 0.061

D) 0.022

9) C

- 10) Find the probability of selecting 12 or more girls.

A) 0.006

B) 0.022

C) 0.007

D) 0.001

10) C

Provide an appropriate response.

- 11) In a game, you have a  $\frac{1}{27}$  probability of winning \$100 and a  $\frac{26}{27}$  probability of losing \$4. What is your expected value?

A) -\$0.15

B) \$7.56

C) \$3.70

D) -\$3.85

11) A

- 12) A 28-year-old man pays \$200 for a one-year life insurance policy with coverage of \$120,000. If the probability that he will live through the year is 0.9994, what is the expected value for the insurance policy?

A) -\$128.00

B) \$72.00

C) -\$199.88

D) \$119,928.00

12) A



Determine whether the given procedure results in a binomial distribution. If not, state the reason why.

13) Choosing 6 marbles from a box of 40 marbles (20 purple, 12 red, and 8 green) one at a time with replacement, keeping track of the number of red marbles chosen.

13)

B

A) Not binomial: there are more than two outcomes for each trial.

B) Procedure results in a binomial distribution.

C) Not binomial: the trials are not independent.

D) Not binomial: there are too many trials.

Assume that a procedure yields a binomial distribution with a trial repeated  $n$  times. Use the binomial probability formula to find the probability of  $x$  successes given the probability  $p$  of success on a single trial. Round to three decimal places.

14)  $n = 5, x = 2, p = 0.70$

A) 0.198

B) 0.132

C) 0.700

D) 0.464

14)

B

15)  $n = 14, x = 3, p = 0.5$

A) 0.022

B) 0.029

C) 0.033

D) 0.125

15)

A

Find the indicated probability. Round to three decimal places.

16) The participants in a television quiz show are picked from a large pool of applicants with approximately equal numbers of men and women. Among the last 11 participants there have been only 2 women. If participants are picked randomly, what is the probability of getting 2 or fewer women when 11 people are picked?

A) 0.006

B) 0.033

C) 0.032

D) 0.027

16)

B